

Students' Ability To Think Mathematically in Solving PISA Mathematics Problems Content Change and Relationship

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Abstract

This article describes students' mathematical-thinking process in solving PISA-model mathematics -problems. Mathematical-thinking is not only important for academic succes, but is also essential for developing mathematical reasoning and critical thinking habits in sustainable development for creating better life. In order to achieve the research aims, we used a qualitative approach with descriptive methods. Data were collected using mathematical activities adapted from PISA items. Three students' works that represent each level of mathematical ability were selected for in-depth analysis. The findings shows that students' mathematical-thinking ability in solving PISA-model mathematics-problems is determined by how the students go beyond phases of mathematical-thinking proses (Entry, Attack, Review). Students can solve problems correctly if they are able to go through all phases, although not all aspects of each phase are fulfilled. If students fail in the Entry phase, it is certain that they cannot go through the next two phases properly (Attack and Review).

Keywords: Mathematical thinking; PISA items; Change and Relationship

INTRODUCTION

In order to develop students' mathematical abilities and enhance their performance, mathematics education should not only emphasize on mastery mathematics content, but it should also help students developing their ability in thinking mathematically (Breen & O'Shea, 2010). This is in accordance with the goal of mathematics education in Indonesian, that is preparing students with mathematics understanding and develop students' ability to think mathematically (Anwar, Budayasa, Amin, & de Haan, 2012). Moreover, the ability to think mathematically is one of the literacy needed to build reasoning and critical thinking habits in sustainable development for the creation of a better life (SEAMEO-RECSAM, 2017). Thus, the ability to think mathematically is not only important for academic life, but it is also needed to support our role in social life.

Apart from the importance of students' ability in thinking mathematically, several previous studies have shown that mathematics learning in Indonesia has not been sufficient in helping students to develop their mathematical thinking skills (Anwar et al., 2012; Khamidah & Suherman, 2016; Fatimah, Muhsetyo, & Rahardjo, 2019). This is due to the learning activities at schools mostly about solving problems that needs routine procedures, students' proficiency in routine algorithms or counting ability, and it is declared that students understand

mathematics if they are able to memorize and apply mathematical formula (Anwar et al., 2012; Sepeng, 2013; Devlin, 2019; Satiti & Verdianingsih, 2019). Whereas, the ability to think mathematically emphasizes on cognitive processes, not just being proficient in routine algorithms or arithmetic (Fatimah et al., 2019).

One of the mathematical activities that support students' ability in thinking mathematically is PISA mathematics problems. Cognitive processes in solving PISA problems occur through the activity of formulate real-world contexts into mathematical problems, employ concepts, facts, procedures and mathematical reasoning in solving mathematical problems, and interpreting mathematical solutions obtained into the context of problems and re-examining whether these solutions are suitable for the given the context (OECD, 2014; Stacey, 2015). However, based on the results of the 2018 PISA assessment, performance of Indonesian students in mathematics was less than satisfactory. The performance of Indonesian students is still at level 1 and is in 17th place out of 20 countries that are at level 1 (OECD, 2019). Based on PISA assessment, achievements of each participating country are classified into six levels, in which level 6 is the highest achievement. This shows that Indonesian students' ability in thinking mathematically is still low compared to students' from other countries. Therefore, it is crucial to increase the use of PISA-like mathematics problems as learning activities in order to facilitate the development of students' ability in thinking mathematically.

PISA mathematics problems that focus on mathematical literacy are classified d based on content, context, and process category (OECD, 2013; Stacey, 2015). Content in PISA problems is interpreted as a mathematical structure and topic that underlies a given problem or situation (Stacey, 2015). One of contents in PISA mathematics problems is Change and Relationship (OECD, 2013). Change and Relationship content can be found in various contexts or situations, such as social arithmetic, function, and calculus (Stacey, 2015). However, many students have difficulty in solving PISA mathematics problems with Change and Relationship content, especially in building an understanding of the mathematical concepts that underlie the problems so that the students find it difficult to solve or answer the given questions (Nasriadi & Sari, 2017). The same result was also found in a study conducted by Pratiwi, Trapsilasiwi, Oktavianingtyas, Sunardi, & Murtikusuma (2019) which showed that students' ability in solving PISA mathematics problems in Change and Relationship category was still low, in which the majority of the students were at level 2. Therefore, it is necessary to conduct a study to determine students' ability in thinking mathematically and the process of it in solving PISA-like mathematics problems in Change and Relationship category. This is important because by understanding students' ability in thinking mathematically and the process of it, teachers and educators will be able to determine which aspects of students' mathematical thinking process which are needed to be maintained and which aspects should be improved so that students can solve mathematics problems properly and correctly.

Cognitive skills employed in solving PISA mathematics problems (OECD, 2014; Stacey, 2015) aligned with framework of thinking mathematically initiated by Mason, Burton, & Stacey, (2010). Therefore, in this study researcher applies thinking mathematically framework initiated by (Mason et al., 2010) as a

foundation for conducting the study. The fundamental process of thinking mathematically are specializing and generalizing (Mason et al., 2010). This process integrates into three phases of thinking mathematically; (1) Entry phase (what do I know, what do I want, what can I introduce) – that is how do students get information / facts and make a plan in order to solve the mathematics problems, (2) Attack phase (try, maybe, why), and (3) Review phase (check, reflect, extend) - that is when results are obtained and to be checked. The three phases are related to each other and it is possible that the flow of the three phases is back and forth, which means that working on a phase is very likely to return to the previous phase or maybe it will jump to the final phase (Fatimah et al., 2019). The relationship between process and phases in thinking mathematically can be seen in Figure 1 below.

Figure 1. Process and Phases of Thinking Mathematically

Based on the explanation above, this study is aimed to obtain a detailed and comprehensive understanding of students' ability to think mathematically (mathematical thinking) and the process of it in solving PISA-like mathematics problems category Change and Relationship.

RESEARCH METHOD

The method employed in this study is qualitative approach with descriptive methods (Sugiyono, 2018). This study is aimed to obtain a detailed and comprehensive understanding of students' ability to think mathematically (mathematical thinking) and the process of it in solving PISA-like mathematics problems. The study was conducted on 30 students of class IX at MTs Salafiyah Syafi'iyah Jombang.

Data collection began with students working on the PISA-like mathematics problem (Figure 2). The results of the students' work were then examined. Based on the results of this examination, students' works were arranged into three categories. The mathematics problem comprised of two sub questions. The arrangement of students' work was carried out based on the number correct answers. The three categories are PD-K1, PD-K2, and PD-K3. PD-K1 means a student is not able to answer correctly all of the two sub questions; PD-K2 means a student is able to answer correctly one question only, and PD-K3 means a student answer both sub questions correctly.

From each category, there would be selected one students' work randomly for in-depth analysis. The students whose work were selected hereinafter referred to as selected subjects. In-depth analysis was carried out against the frameworks of thinking mathematically initiated by Mason et al. (2010). This in-depth analysis was supported by deep interviews. Thus, the data collected in this study were students' work and recordings of deep interview.

The researchers composed a rubric of thinking mathematically that contained indicators adapted from the thinking mathematically framework initiated by Mason et al. (2010). This rubric was employed as a guideline for analyzing

students' mathematical thinking processes and their ability to think mathematically. This rubric was also employed as deep interview guideline to detect indicators that arose from each aspect and phase of thinking mathematically.

Table 1. Rubric of Thinking Mathematically

Phase	Aspect	Code	Indicator
Entry	. I know	1.a.1	Read the question carefully
		1.a.2	Specialize to discover what is involved
		1.a.3	What ideas/skils/facts seem relevant to be applied in solving the problems
		1.a.4	Do I know any similar or analogous questions
	. I want	1.b.1	Classify and sort information
		1.b.2	Be alert to ambiguities
		1.b.3	Specialize to discover what the real question is and what should be determined/found
	. Introduce	1.c.1	Represents facts and information that is known from the problem into pictures, diagrams or symbols
		1.c.2	Organizing known facts and information from problems, representing them in mathematical notation.
	Attack	. Try dan Maybe	2.a.1
2.a.2			Check the guesswork (allegations) through the process of problem solving / answering questions from the problems
. Why		2.b.1	Check whether the guesswork (allegations) is correct or wrong
		2.b.2	If the allegations submitted are wrong, then the allegations can be rejected, or how to modify the allegations so that they are true
		2.b.3	Confirming by providing logical reasoning related to acceptance or rejection of an allegation
Review	. Check	3.a.1	Check on calculation
		3.a.2	Check on formula or procedure employed to ensure that the techniques are appropriate
		3.a.3	Check that the resolution fits the questions and context of the problems
	. Reflect	3.b.1	Reflect on key ideas on the problem solving process
		3.b.2	Reflect on the resolution; can it be made clear
	. Extend	3.c.1	Extend the result to a wider context by generalizing
		3.c.2	Extend by seeking a new path or different technique in solving problems or answering questions.

In order to make it easier to refer to a certain indicator in carrying in-depth analysis, indicator codes were used as shown in Table 1 above.

The following are mathematical activities used in this study. Mathematical activities are adapted from PISA mathematics problems category Change and Relationship (OECD, 2013).

Belanja Nadia

Nadia baru saja pindah ke Surabaya. Ia mendapat pekerjaan baru di kota Pahlawan tersebut. Nadia ini membeli beberapa peralatan dapur yang belum ia miliki, seperti halnya teflon, blender, dan panci. Nadia akan membeli barang-barang tersebut di toko *Adinda Hardware*.

Berikut ini daftar harga (harga jual normal) teflon, panci dan blender di toko *Adinda Hardware*

Figure 2a. PISA-like mathematics problems



- a. Hari ini ada diskon di toko *Adinda Hardware*. Untuk pembelian dua atau lebih barang akan ada potongan 20% dari harga normal. Jika Nadia hanya membawa uang Rp. 200.000,- untuk belanja peralatan dapur, tentukan apakah pembelian pada tabel di bawah ini dapat dilakukan oleh Nadia!
- b. Harga jual normal Blender, Teflon Awet dan Panci Kuwat sudah termasuk laba 37,5% . Harga barang tanpa laba disebut harga Grosir. Laba dihitung dalam bentuk persen dari harga Grosir. Berdasarkan informasi di atas, tentukan apakah formula-formula berikut menunjukkan hubungan yang benar antara harga jual normal (J) dengan harga grosir (G)! Uraikan alasan kalian!

Belanja	Dapatkah Nadia membeli peralatan tersebut dengan uang Rp. 200.000,-
Blender dan Teflon	Dapat/Tidak
Blender dan panci	Dapat/Tidak
Ketiga peralatan di atas, blender, teflon dan panci	Dapat/Tidak

Formula	Apakah formula berikut ini benar?
$J = G + 0,375$	Benar/Salah
$G = J - 0,375J$	Benar/Salah
$J = 1,375G$	Benar/Salah
$G = 0,625J$	Benar/Salah

Figure 2b. PISA-like mathematics problems

RESULTS AND DISCUSSION

There were three selected students' work for in-depth analysis. Researchers determined several references in analyzing the work of selected subjects. A subject (student) was declared successful in one phase of mathematical thinking if his/her works fulfilled all aspects of the related phase and the answer was correct. A student was declared to have fulfilled an aspect of mathematical thinking if his/her works met at least one indicator of each related aspect. Following are the results and discussion of the selected students' works.

Students' thinking mathematically in solving sub activity/question "A"

PD-K2 and PD-K3 have answered sub-questions-A correctly. Meanwhile, PD-K1 could not complete this activity properly. The following is the work of the PD-K1.

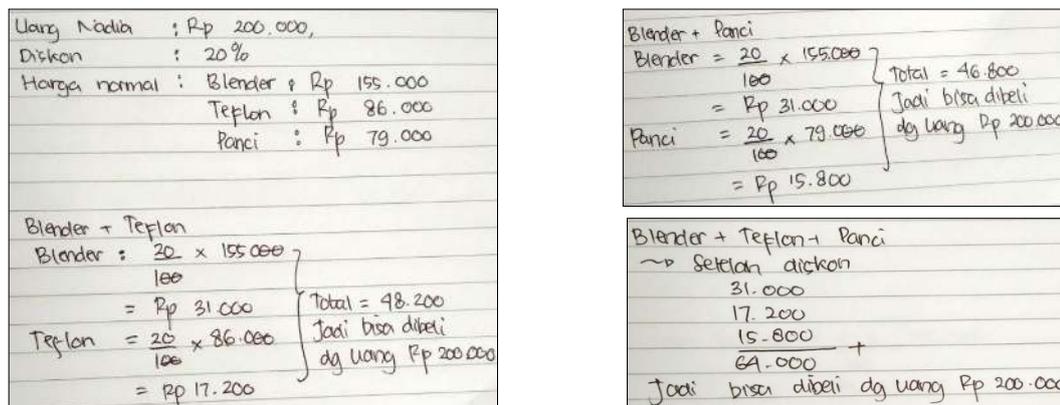


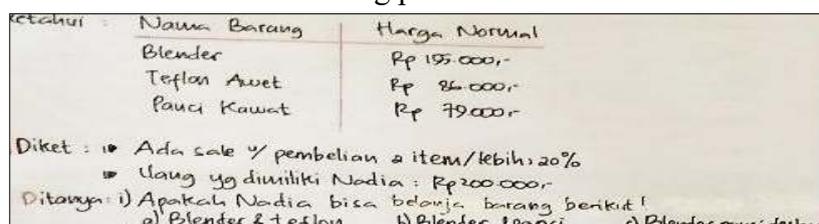
Figure 3. PD-K1's works on sub activity/question-A

As shown in Figure 3, PD-K1 wrote relevant information in order to answer the questions. This shows that PD-K1 has fulfilled indicators 1.a.1 and 1.a.2 in **I know-aspect**. Then, PD-K1 determined 20% of the normal price of each item. However, PD-K1 made a mistake in determining the price of goods after discount. PD-K1 assumed that 20% of the normal price was the item price after discount. In fact, 20% of the normal price is the discount value. This shows that PD-K1 did not understand the meaning of discount and the relationship between normal selling price, discount value and goods price after discount. PD-K1 did not understand correctly what the question was about. Thus PD-K1 has failed to go through the Entry phase properly.

Until the end of problem solving process, PD-K1 still comprehended 20% of the normal price as the price of the goods after discount. This shows that PD-K1 did not re-check or reflect on whether the concept applied was correct or not. So that PD-K1 failed in the Attack phase. It also shows that PD-K1 did not do the Check or Reflect aspects in the Review phase. In addition, it seems that the PD-K1 only did computation on the numbers listed in he problem without realizing the mathematics concepts should be applied.

Referring to the PD-K1's work above, the failure in the Attack and Review phase was caused by PD-K1 failing in the Entry phase, especially PD-K1 could not meet the indicator 1.b.3 in **I want-aspect**. Even though the calculation results are correct, these results are not the right solution for the given problem. Thus, it can be concluded that the failure in the Entry phase might hinder students to go through the next two phases properly and correctly.

PD-K2 and PD-K3 were able to do sub activity/question A correctly so that they got the right answer/solution. Following are PD-K2 and PD-K3' works and the analysis of their mathematical thinking processes.



✓ Pernyataan 2 .
Blender + Panci = Rp. 124.000 + Rp. 63.200
= Rp. 187.200
Nadia dapat melakukan pembelian , karena Rp. 187.200 < Rp. 200.000 .

✓ Pernyataan 3 .
Blender + teflon + Panci = 124.000 + 68.000 + 63.200
= Rp. 256.000 .
Nadia tidak dapat melakukan pembelian , karena 256.000 > 200.000 .

Figure 5. PD-K3's works on sub activity/question-A

As shown in Figures 4 and 5, PD-K2 and PD-K3 wrote information derived from the problem correctly, completely and relevant. Both PD-K2 and PD-K3 have met indicators 1.a.1 and 1.a.2 in **I know-aspect**. PD-K2 also classified the informations and sorted them into "Diketahui" and "Ditanya". This shows that PD-K2 has fulfilled indicator 1.b.1 in **I want-aspect**.

Furthermore, PD-K2 and PD-K3 had determined the discount value of 20% from the normal price. PD-K2 and PD-K3 understood the concept of discount correctly. Therefore they had determined the price of the goods after discount (purchase price) by subtracting the normal price from the discount value (in rupiah). Thus, the PD-K2 and PD-K3 have fulfilled indicator 1.b.3 in **I want-aspect**. PD-K2 also organized the facts / information and what to look for into a table. This shows that the PD-K2 has fulfilled the **Introduce-aspect**. Whereas in PD-K3's works, the **Introduce-aspect** appears in the use of the inequality sign to compare the price of goods after a discount with Nadia's money. This shows that PD-K3 has met indicators 1.c.1 and 1.c.2 in **Introduce-aspect**.

Based on PD-K2 and PD-K3's works above, it can be deduced that PD-K2 and PD-K3 have fulfilled all aspects of the **Entry-phase**. PD-K2 and PD-K3's solutions and answers are also correct. Thus, PD-K2 and PD-K3 have gone through the **Entry-phase** properly and successfully.

In the next section, both PD-K2 and PD-K3 examined which purchases that Nadia could make with Rp. 200,000,-. This shows that PD-K2 and PD-K3 have met indicator 2.a.2 in **Try and Maybe-aspects**. Both PD-K2 and PD-K3 have always provided a clear and logical reasoning to support their arguments on whether a purchase could be made or not. Thus, PD-K2 and PD-K3 have fulfilled indicators 2.b.1 and 2.b.3 in **Why-aspect**. Based on PD-K2 and PD-K3's works, each student has fulfilled all aspects of the **Attack-phase**. PD-K2 and PD-K3's solutions and answers are also correct. Thus, PD-K2 and PD-K3 have gone through the **Attack-phase** properly and successfully.

Both PD-K2 and PD-K3 did the calculation correctly and the computation was also correct. Researchers suspected that the two students have met the indicators 3.a.1 and 3.a.2 in the **Check-phase**. The researcher explored this through the deep interviews, and it was revealed that PD-K2 and PD-K3 had already double-checked their formula, computation and calculation results. Thus, the researchers' allegations have been proved correct.

At the end of the problem solving process, both PD-K2 and PD-K3 clearly wrote down which statements or purchases that Nadia could make. They also provided logical reasoning to support their answers. This shows that PD-K2 and PD-K3 have met indicator 3.b.2 in **Reflect-aspect**. The **Extend-aspect** did not appear in PD-K3's works. But it was emerged in PD-K2's works through the use of different methods in solving the given problem.

NO	NAMA BARANG	HARCA NORMAL (N)	TOTAL	HARCA BELI (80% N)	KET
1	Blender & Teflon	Rp 155.000 + Rp 26.000	Rp 241.000	Rp 192.800	✓
2	Blender & Panci	Rp 155.000 + Rp 79.000	Rp 234.000	Rp 187.200	✓
3	Blender & Teflon & Panci	Rp 155.000 + Rp 26.000 + Rp 79.000	Rp 360.000	Rp 256.000	×

Figure 6. PD-K2's works employing different method

Students' thinking mathematically in solving sub activity/question "B"

Only PD-K3 has answered sub activity/question B correctly. Meanwhile, PD-K1 and PD-K2 could not complete this activity properly. The following are PD-K1 and PD-K2's works.

$J = \text{harga jual normal}$ $G = \text{harga grosir}$ $\text{Labai} = 37,5\%$	$\text{Labai Teflon} = \frac{37,5}{100} \times 88.000$ $= \text{Rp } 32.250$
$\text{Labai Blender} = \frac{37,5}{100} \times 155.000$ $= \text{Rp } 58.125$	$\text{Labai Panci} = \frac{37,5}{100} \times 79.000$ $= \text{Rp } 29.625$

Figure 7. PD-K1's works on sub activity/question -B

Diket: Didalam harga normal termasuk laba 37,5%
 1) Harga grosir: harga barang tanpa laba
 2) J: harga jual normal
 3) G: harga grosir.

Ditanya: i) Keseluruhan formula:
 a) $J = G + 0,375G$ c) $J = 1,375G$
 b) $G = J - 0,375J$ d) $G = 0,625J$
 ii) Alasan.
 iii) Hitungan.

Penyelesaian:
 Dari beberapa hal yg telah diketahui, maka bisa disajikan dalam tabel berikut!

NO	NAMA BARANG	HARCA NORMAL (J)	LABA 37%	HARCA GROSIR (G)
1	Blender	Rp 155.000,-	Rp 58.125,-	Rp 96.875,-
2	Teflon Awet	Rp 26.000,-	Rp 32.250,-	Rp 53.750,-
3	Panci Kuwat	Rp 79.000,-	Rp 29.625,-	Rp 49.375,-

Figure 8a. PD-K2's works on sub activity/question -B

Uji Coba Formula 1
 $J = G + 0,375G$
1) $155.000 = 96.875 + 0,375 \cdot 96.875$
 $155.000 \neq 96.875,375$
2) $86.000 = 53.750 + 0,375 \cdot 53.750$
 $86.000 \neq 53.750,375$
3) $79.000 = 49.375 + 0,375 \cdot 49.375$
 $79.000 \neq 49.375,375$
TIDAK SESUAI

Uji Coba Formula 2
 $G = J - 0,375J$
1) $96.875 = 155.000 - 0,375 \cdot 155.000$
 $96.875 = 155.000 - 58.125$
 $96.875 = 96.875$
2) $53.750 = 86.000 - 0,375 \cdot 86.000$
 $53.750 = 86.000 - 32.250$
 $53.750 = 53.750$
3) $49.375 = 79.000 - 0,375 \cdot 79.000$
 $49.375 = 79.000 - 29.625$
 $49.375 = 49.375$
SESUAI

Uji Coba Formula 3
 $J = 1,375G$
1) $155.000 = 1,375 \cdot 96.875$
 $155.000 \neq 133.203,125$
2) $86.000 = 1,375 \cdot 53.750$
 $86.000 \neq 73.906,25$
3) $79.000 = 1,375 \cdot 49.375$
 $79.000 \neq 67.890,625$
TIDAK SESUAI

Uji Coba Formula 4
 $G = 0,625J$
1) $96.875 = 0,625 \cdot 155.000$
 $96.875 = 96.875$
2) $53.750 = 0,625 \cdot 86.000$
 $53.750 = 53.750$
3) $49.375 = 0,625 \cdot 79.000$
 $49.375 = 49.375$
SESUAI

Figure 8b. PD-K2's works on sub activity/question –B

In Figures 7 and 8, PD-K1 and PD-K2 wrote down information derived from the given problem. PD-K1 and PD-K2 have met indicators 1.a.1 and 1.a.2 in **I know-aspect**. PD-K2 classified the information into “Diketahui” and “Ditanya”, so that PD-K2 has fulfilled indicator 1.b.1 in **I want-aspect**. The **Introduce-aspect** appeared in PD-K2's work through the use of table in organizing informations (what is known).

It can be seen on PD-K1 and PD-K2's works, each student determined the profit of sales by multiplying the percentage of profit (37.5%) with the normal selling price (J). However, the questions states that profit is calculated from the wholesale price (G). Therefore, PD-K1 and PD-K2 did not understood correctly what the problem was about, and unfortunately they could not go through the Entry phase properly.

PD-K1 did not continue the process of solving problem (Figure 7). PD-K1 discontinued the works at determining the value of profit in which PD-K1 also made a mistake. Based on deep interviews with PD-K1, it was obtained that PD-K1 did not continue the work due to the confusion about what to do. This strengthen the evidence that PD-K1 did not understand the questions. Meanwhile, PD-K2 tried to check the formula (Figure 8). However, due to the incorectly applied of mathematics concept of profit, the result obtained could not be used as a correct and logical reasoning in answering the question. In addition, this also shows that PD-K2 did not do double-check on mathematical concepts wether it was correct or not. Therefore, PD-K2 has failed in **Check-aspect** in the Review phase.

Based on PD-K1 and PD-K2's works, it can be deduced that the failure of PD-K1 and PD-K2 in the Entry phase initiated their failure the next two phases, Attack and Review phases. Thus, it can be concluded that students' work on the Entry phase will affect the problem-solving process in the next two phases.

PD-K3 is the only student who is able to solve sub-question-B correctly and properly. Moreover, PD-K3 has obtained correct solution/answer. The following is PD-K3's work and it's analysis.

Handwritten work by PD-K3:

Harga barang tanpa laba = Grosir
 $J - L = \text{Grosir}$
 Laba dihitung dalam bentuk % dari harga grosir
 $L = 37,5\% \times G$
 $= \frac{37,5}{100} \times G$
 $= 0,375 G$
 $\text{Grosir} = J - L$
 $G = J - 0,375 G$

Dari uraian tersebut dapat disimpulkan formula:

- $J = G + 0,375 G$ adalah **salah** karena
 $J - 0,375 G = G$
 $J = G + 0,375 G$
- $G = J - 0,375 J$ adalah **salah** karena
 seharusnya laba dihitung dan persen dari harga
 grosir bukan harga jual. yaitu $G = J - 0,375 G$
- $J = 1,375 G$ adalah **benar**. Di dapat
 dari $J = G + 0,375 G$
 $= 1,375 G$.
 karena memiliki variabel sama
 jadi dapat dijumlah
 dg G berilai 1
- $G = 0,625 J$ adalah **salah** karena dari
 pernyataan yg ada : $G = J - 0,375 G$

Figure 9. PD-K3's works on sub activity/question –B

It can be seen in Figure 9, PD-K3 wrote relevant information to solve the questions/problems. It means that PD-K3 has fulfilled indicators 1.a.1 and 1.a.2 in the **I know-aspect**. PD-K3 also represented the relationship between selling price (J), wholesale price (G), and profit (L) into a mathematical equation $J - L = \text{Wholesale (G)}$. This shows that PD-K3 was aware of mathematics concepts underlying the questions. So that, PD-K3 has fulfilled indicator 1.a.3 in the **I know-aspect**. Then, PD-K3 determined the profit by multiplying the profit percentage 37.5% with the wholesale price (G). This is true, so that the PD-K3 has met indicator 1.b.3. in the **I want- aspect**. Through a simple operation, PD-K3 has obtained an equation for the value of profit, that is $L = 0.375G$. PD-K3 also expressed most of information derived from the problem into mathematical equations and notations. This shows that PD-K3 has fulfilled the indicators 1.c.1 and 1.c.2 in the **Introduce-aspect**. PD-K3 has fulfilled all aspects in the Entry phase and PD-K3's works are also correct. Thus, PD-K3 has gone through the Entry phase properly and successfully.

In the next section, PD-K3 determined which formula states the correct association for wholesale price (G), normal selling price (J) and profit percentage (L) 37.5%. This shows that PD-K3 has fulfilled indicator 2.a.2 in the **Try and Maybe-aspects**. PD-K3 has provided logical and appropriate reasoning to support the answers, so that PD-K3 has also fulfilled indicators 2.b.1 and 2.b.3 in the **Why-aspect**. Therefore, PD-K3 has fulfilled all aspects of the Attack phase and PD-K3's works are also correct. So that PD-K3 has been through the Attack phase properly and correctly.

PD-K3 also did the calculation correctly and obtained correct result. Researchers suspected that PD-K3 has met indicators 3.a.1 and 3.a.2 in the Check phase. The researcher verified this assumption during deep interview. Based on the deep interview it was revealed that PD-K3 had rechecked the formula, calculation and the result obtained from the computation. Therefore, the researchers' allegations has been proved correct.

In PD-K3's works, the student clearly stated which formula shows the correct relationship between wholesale price (G), normal selling price (J) and profit (L). This indicates that PD-K3 has met indicator 3.b.2 in the **Reflect-aspect**. In PD-K3's works, there is no **Extend-aspect** emerged. However, PD-K3 has fulfilled the **Check and Reflect-aspects** of the Review phase properly.

Based on the discussion above, it can be seen that students' failure in initial stage of the mathematical thinking process, that is Entry phase, might impede students' performance in the next two phases (the Attack and Review phase). This will inhibit students' ability to answer the questions and solving the problems. This is in accordance with the findings of Fatimah et al. (2019) which reveals that students can go through the Attack and Review phase if the student is able to go beyond the Entry phase first.

In addition, students are able to solve problems properly and correctly after they are able to complete all phases, although not all of aspects of each phase are fulfilled. However, it can be seen that indicator 1.b.3; "Specialize to discover what the real question is and what should be determined/found ", in **I want-aspect**, determines whether the students will be able to answer the questions and solve the problem correctly or not. This is in line with the results of study conducted by Wijaya, van den Heuvel-Panhuizen, Doorman, & Robitzsch (2014) which shows that difficulties in answering mathematical problems are generally caused by the difficulty in understanding the problem or the purpose of the problem itself.

As shown in PD-K1's work on sub-question-A, PD-K1 only performed computation on numbers listed in the problems. PD-K1 multiplied the 20% discount with the normal price (J) and interpreted the result as price of the item after the discount. Supposedly, the price of the item after the discount is $= J - 20\%J$. Several previous studies have also shown similar case, that is in solving mathematics problems students tend to focus on computing the given number only without realizing and employing the relevant concepts which underlies the problem (Sepeng, 2013; Nasriadi & Sari, 2017; Satiti & Verdianingsih, 2019).

CONCLUSION

Based on the discussion in the above section, it can be concluded that: 1) The students' ability to think mathematically in solving PISA-like mathematics problems is determined by how students go beyond the phases of the mathematical thinking process (Entry, Attack, Review phase). 2) Students can go through the Attack and Review phase if the students are able to go beyond the Entry phase first. If students fail in the Entry phase, it can be ascertained that these students are not

able to go through the next two phases (Attack and Review phases). 3) Students are able to solve problems properly and correctly after they are able to go through all phases, although not all aspects of each phase are met. However, the indicator "Specialize to discover what the real question is and what should be determined/found", in the I want-aspect determines whether the students are able to solve the problem or not. 4) In solving mathematics problems, there are still many students who only do calculation on numbers without comprehending the mathematics concept underlying the problem.

This research employs results of PISA assessment as a basis and uses mathematics problems adapted from PISA items as an instrument. PISA-like mathematics problems are not the only activity to determine students' abilities and performances in mathematics. Therefore, in further research, the broader scope must be taken into account to determine the students' mathematical thinking process and ability. In addition, only Change and Relationship content was selected from PISA items. So that the discussion and analysis of the students' mathematical thinking process is limited to the content used in the mathematics problem. Further research can be developed for other content and contexts on PISA mathematics problems.

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