

Student Thinking Process in Solving Mathematical Representation Problems

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Corresponding author:	Abstract
Alfiah Nurfadhilah AM. Hindi, alfiah.nurfadhilah.am.hindi@unpatompo.ac.id	This research was qualitative research with descriptive approach, which aimed determined the level of success of students in solving representation problems based on the type of representation, namely numbers, algebra, geometry, and statistics and to know their thinking processes. The subjects of this study were first semester students at Patompo University who were selected using a purposive sampling technique. The subjects of this study were 2 students in each representational domain. Data were analyzed to determine the results of problem solving and students' thinking processes in solving representation problems in mathematics. Based on the results of the research on solving visual representation problems based on TRM-01 and TRM-02, most of the students failed especially in algebraic material which was caused by difficulties in solving representation problems. This is shown based on the percentage of students' success in solving representation questions.
Keywords: thinking process; problem-solving; graphics representations	

Hindi, A., N., AM, & HR, I., S.. (2023). Student Thinking Process in Solving Mathematical Representation Problems. *Mathematics Education Journal*, 7(1), 47-59. DOI: 10.22219/mej.v7i1.24830

INTRODUCTION

The activity of how individuals obtain information, how that information is represented and transformed into knowledge, how that knowledge is stored into memories which are later recalled, and how that knowledge is used to take actions or decisions, all of which are mental activities or thinking activities or thinking processes. Mental processes or thinking processes are processes that humans use to learn to acquire knowledge by understanding and conceptualizing problem situations, making hypotheses and how to solve problems, controlling and improving problem-solving processes (Hindi & HR, 2020).

One of the mental activities is representation. Why is this representation interesting to study as part of a learning model? To begin with this representation, the author tries to remember childhood or simple observation results in children who are still at the Piaget version of pre-operational mental development. Representation is defined simply by Mainali (2021) as a sign or a combination of signs, characters, diagrams, objects, images or graphics. Beyond this definition, representation involves translating problems or new forms, translating diagrams or physical models into symbols or words, representations are also used in translating or analyzing verbal problems to clarify meaning (NCTM, 2000). Meanwhile Goldin & Shteingold (2001) defines in more detail as a configuration (shape or

arrangement) that can describe, represent, or symbolize something in a way that is categorized into two stages, namely internal representations and external representations.

Students are taught how to communicate by reading and writing (literacy) and learning numbers (numeration). However, the art of combining these two skills is often overlooked. Representation, which can be defined as the ability to communicate numerical visual information is still neglected. Most students only have skills in reading, writing and arithmetic even though translating graphical language is an important aspect of learning Mathematics (Zhao & Gaschler, 2022).

Graphs are one of the most common mathematical functions used to display information. They are represented in text books, standardized tests, as well as other print and electronic media in classrooms (Mukherjee et al., 2021; Priti & James, 2002; Shreiner & Guzdial, 2022). Visual representation is one of the most commonly used mathematical functions to display information represented in textbooks, standardized tests, and other print and electronic media.

Several developed countries such as Australia, England and the United States have paid special attention to graphical knowledge such as diagrams, graphs and symbols for students' abilities included in the mathematics learning curriculum. The ability of students who specifically emphasize the ability to interpret graphs for various grade levels in schools (Lowrie & Diezmann, 2007).

An individual's ability to interpret a visual representational language is based on the perception and concept of representation. There are various ways students interpret the graphical language of mathematics. Giving answers is not necessarily in accordance with the concepts that have been taught so it is necessary to trace the thinking processes of students in solving mathematical representation problems. In general, students represent problems more to the mastery of knowledge and experience possessed by each student (Lowrie & Diezmann, 2009).

According to Krathwohl (2002), there are three aspects required for someone to understand information through visual information. The three aspects in question are the aspect of translation (translation), namely the ability to read, capture a meaning through graphics then expressed in another way based on the original statement, the aspect of interpretation (interpretation), namely the ability to conclude, then expressed based on existing data in his view into another form, as well as aspects of prediction (extrapolation), namely one's ability to make predictions or predict trends based on observed data.

Baker et al. (2001) suggests "*typically, students' ability to decode particular types of graphics is not the focus in mathematical tests. However, because knowledge of graphics impacts on mathematical performance, we constructed an instrument to assess students' knowledge of graphics*". In particular, students' ability to represent each type of representation was not focused on the math test, but graphic knowledge had an effect on graphical math performance. Several forms of representation—such as diagrams, graphs, and symbolic expressions—have long been an integral part of learning mathematics in schools. Unfortunately, these forms of representation are taught directly, as if they were a learning objective. This approach limits the power and usefulness of representation as a tool for learning, working, and thinking mathematically.

Seeing the importance of representation in learning mathematics, it is important to encourage students to represent their mathematical ideas in a way they understand, even if the representation is not common, unconventional (different from the others). At the same time, students also need to learn forms of representation that are common (conventional) to be able to facilitate them in learning mathematics and their communication with other students in mathematical ideas. In addition, for today, the integration of technology in further mathematics learning can increase the need for students to adapt comfortably in using new mathematical representations (Pedersen et al., 2021). Mathematical representation is an important part of educational goals in mathematics. Representation in solving mathematical problems can help students to organize, present, look for relationships and evaluate a set of data so that it supports students to interpret and make sense of the information provided (Goldin & Shteingold, 2001). In this case, the teacher's role is to help students express how the process goes on in their minds when solving problems, for example by asking students to tell the steps that are in their minds. This is necessary to find out thinking errors and tidy up the knowledge network of students. The thought process is an activity that occurs in the human brain. Incoming information and data is processed in it, so that what is already inside needs adjustments, even changes (HR & Hindi, 2020). This process is called adaptation.

The thought process according to Solso et al. (2014) includes three main components, namely: (1) thinking is a cognitive activity that occurs in one's mental or mind, invisible, cannot be concluded based on visible behavior, (2) thinking is a process that involves some manipulation of knowledge in the cognitive system, knowledge stored in memory is combined with current information so as to change one's knowledge of the situation at hand, and (3) thinking activity is directed to produce problem solving.

According to Piaget (in Santrock, 2018) information transformation can be carried out in two ways, namely: (1) assimilation, namely changing the structure of information that has just entered short-term memory to match the existing scheme in long-term memory, and (2) accommodation, namely changing the existing schema in long-term memory to suit the structure of the newly entered information, so that the new information can be received, meaning that it can be stored in long-term memory.

To be able to achieve this, the researcher wants to examine more about the thinking process in solving mathematical representation problems in early semester students of Mathematics Education Study Program at Universitas Patompo. Indicators used to see the process of thinking based on Piaget's theory of assimilation and accommodation processes.

RESEARCH METHOD

Types of Research

This type of research is qualitative research with a descriptive approach. This study analyzes students' problem-solving abilities in visually representing mathematical problems and students' abilities in solving mathematical problems based on the visual information presented.

Research Subject

This research was conducted in the first semester students of mathematics education study program at Universitas Patempo by conducting a mathematical representation test in the form of essay questions related to visual/graphic information. This step is intended to determine the level of success of students in solving problems representing and interpreting the visual information presented. The results of the mathematical representation problem solving tests were analyzed to determine which subjects would be selected in the study. Selection of subjects using purposive sampling technique. the subject is taken with a specific purpose or goal because the researcher assumes that the subject has the information needed in the research. The number of students in this study were 40 students on the Mathematical Representation Test (TRM-01) and 35 students on the Mathematical Representation Test (TRM-02). Next, to determine several research subjects to be interviewed.

Research Procedure

Based on the background and research objectives as in the previous chapter, the research is focused on:

1. The level of success in solving representation problems
2. The process of thinking in solving problems related to assimilation and accommodation
3. Material related to mathematical representation

The flow chart of the research implementation procedure is shown in Figure 1 as follows.

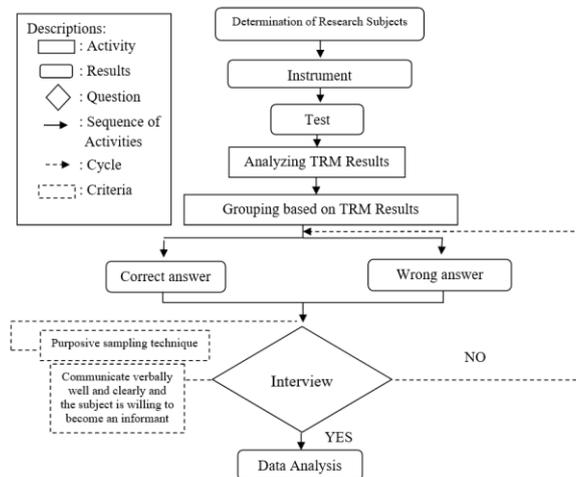


Figure 1. Flowchart of Research Implementation Procedures

Data Collection Technique

Data collection techniques used in this study are as follows:

1. Representation Test

To collect data about the description of students' representational problem-solving abilities, a representation test was made in which students were asked to solve problems visually and verbally. This test is prepared by taking the following steps.

- a. The tests were made in two types, namely in the form of verbal (TRM-01) and in the form of visual (TRM-02) related to material in class VII, namely the domain of numbers, algebra, geometry and statistics.
 - b. Perform expert validation as well as expert consultation
2. Interview
Interviews in this study were used to reveal things that were unclear or unclear in students' written work. In this study, semi-structured interviews were used where the interviewer would first ask a series of structured questions, then deepen one by one in order to obtain further information. Thus, complete and in-depth information can be obtained.
3. Documentation
Documentation is a method for obtaining or knowing something in the form of photographs and video recordings during research, as well as the identity of research subjects obtained from lecturer data.

As for the data analysis flowchart about students' ability to visual representation of mathematics, shown in Figure 2 as follows.

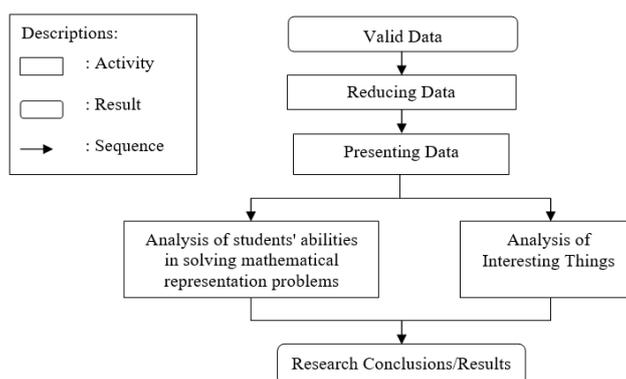


Figure 2. Flow Chart of Data Analysis Techniques.

RESULTS AND DISCUSSION

Based on the research results of visual representation problem solving based on TRM-01 on 40 students. Particularly in the matter of numbers from 40 students, as many as 27 (67.5%) students gave the correct solution and 13 (32.5%) students still had difficulty in representing visually. In the algebraic domain, only 11 out of 40 (27.5%) students gave the correct representation completion. Furthermore, for the geometry domain where students were asked to visualize the meaning of verbal questions, 17 students (42.5%) correctly completed the given representation problem and 1 student did not answer at all. As for the statistical domain, out of 40 students, 25 students (62.5%) gave the correct answer and 2 students did not answer at all. Almost all students are able to describe bar charts. However, there was an error in calculating the amount of data for a cell phone brand so that the answers obtained were not correct.

Most students still experience difficulties from the four domains of representation. Especially in the form of visual presentation, even though it is correct, it is still not perfect. Most students also do not write down their level of

marbles.

Q : *What is being asked?*

S1 : *How many marbles are in the bag.*

Based on the interview excerpt above, the researcher assumes that the subject is able to understand the problem. This is shown when the subject is able to mention things that are known and asked about representation questions.

Meanwhile, based on the results of the written work, the researcher assumes that the subject has not been able to make a correct settlement plan. This can be seen in written work where the subject in describing a scale appears to be in balance. This is not in accordance with the intent of the question so that the settlement plan indicated by the subject is not correct. The subject has not been able to determine the sign that should be used to solve the problem.

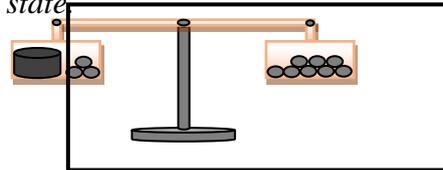
Furthermore, at the stage of problem solving the subject wrote down the process of solving which was not quite right regarding the problem of the representation given. The subject still uses the solution plan to find the answer. In this case the subject does not understand how to make a correct equation. The subject only matched the actual result, which was 8. However, the meaning of the value was still not correct because the subject answered that the contents of the bag were 8. In fact, using the inequality sign meant that the contents of the bag were less than 8. The subject also did not describe the completion graph for this problem.

Furthermore, the results of research on the ability to solve verbal representation problems of 35 students. The ability to solve TRM-02 interpretation of the number domain, as many as 16 out of 35 students (45.7%) gave the correct solution. Furthermore, for the algebraic domain, as in problem number 2 regarding one-variable linear equations, only 10 students (28.6%) were correct and 1 student did not provide a solution. Most of the solutions that are not correct are because students have not been able to translate the meaning of the Figure such as the sign used whether it is an equation or an inequality and do not know how to place which number on the left and right side. But what is interesting about one of the students who has correct completion on the written test in accordance with the aspects assessed based on the scoring guidelines, when being interviewed the subject had experienced an error in the calculations. This is found in solving geometric domain problems where 20 out of 35 students (57.1%) gave the correct solution and 2 students did not answer at all. Furthermore, in question number 4 which measures the ability to interpret statistical domains on a pie chart, 22 out of 35 students (62.9%) gave the correct answer and 1 student did not give the correct answer.

Most students also have not been able to solve visual problems into verbal form, from the four information domains. Especially in terms of writing, in writing answers most students give answers directly without going through the stages of obtaining these answers so that researchers are not sure whether the answers written are fully done by these students. In TRM-02, the algebraic domain also proved to be the most difficult problem for students to solve. Students' difficulties in solving algebraic representation problems occur because students do not have algebraic conceptual knowledge. That one of the indicators in solving problems is the ability to understand problems by recognizing the structural relationships shown through several different representations.

Question 2:

The figure below shows the weight of an iron plate and an iron ball in a balanced state.



An iron craftsman is weighing the two types of iron he has made where the weight of the iron plate is expressed in X kg and the weight of the iron ball is 1 kg. How heavy is the iron plate?

As many as 10 students out of 35 students answered correctly about the representation in the algebraic domain. Most of the students who answered correctly were able to process the given image information such as the state of the scales which were in a state of balance so that they used the equation sign in solving the given representation problem. Furthermore, 25 out of 35 students answered incorrectly. Most of the wrong answers were because students had not been able to translate the meaning of the Figure, such as the sign used, whether it was an equation or an inequality and did not know how to place which number on the left and right side.

The results of written work and interviews with one of the subjects who correctly solved the problem of representation number 2 are as follows Figure 4.

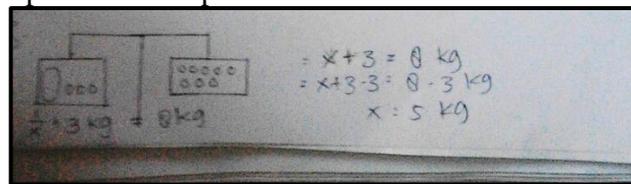


Figure 4. Answer from Subject 2

When the subject was interviewed in more depth regarding the thought process in solving problems, the subject expressed what was in accordance with what he was thinking. The following is an excerpt from an interview with the subject.

Q : Try reading the question

S2 : (read the question)....

Q : What do you think after reading this question?

S2 : About the iron balls and iron plates, ma'am

Q : Why do you think so?

S2 : Because in the question there is a Figure of an iron plate and an iron ball.

Q : When you read the questions, did you immediately understand them?

S2 : You don't understand, ma'am.

Q : Where is the difficulty?

S2 : (Shut up)

Q : What is known from the problem?

S2 : It is known that one iron bag plus 3 iron balls and 8 iron balls next to it.

Q : What is being asked?

S2 : Weight of one bag of iron

Based on the interview excerpts, the researcher assumed that the subject had difficulty understanding the intent of the questions. However, the subject was able to mention the information that was known and was asked in the questions even

though the x value in question was an iron plate, not a bag. In this case, the subject has not been able to concentrate on reading and processing image information properly. So, it can be concluded that the subject has not been able to solve the representation problem properly. Subjects had difficulty in making an equation and looking for the value of the variable. The subject does not yet know how to operate on a value that contains variables. The subject also did not carry out the fourth stage, namely re-checking the answers that had been obtained because the subject knew that the subject had made an error in the completion process and did not know how to correct the answer so that the subject only gave a random solution.

As for the difficulties experienced by students in solving representation problems, namely errors in understanding definitions and variable properties of an algebraic form, errors in interpreting a representation of an algebraic form, not being able to interpret the properties of algebraic forms, errors in calculations, namely addition arithmetic operations, subtraction, multiplication and division of algebraic forms, inability to write work steps in an orderly manner, errors in applying rules, principles or formulas in algebraic operations. Table 1. below shows the level of success of students in solving problems.

Table 1. The success rate of students in solving problems

Representation Test	Domain			
	Number	Algebra	Geometry	Statistics
TRM-01 (against 40 students)	27 students (67,5%)	11 students (27,5%)	17 students (42,5%)	25 students (62,5%)
TRM-02 (against 35 students)	16 students (45,7%)	10 students (28,6%)	20 students (57,1%)	22 students (62,9%)

Discussion

Based on the solutions given by students, the algebraic domain proved to be the most difficult problem for students to solve. Researchers argue that problems in this domain require the solver to get more attention. The results of this study indicate that the majority of students prefer numerical and verbal representations, and a minority of students prefer algebraic representations. These results may be related to students' difficulties in abstract algebra and the way algebra is taught at school. This is in accordance with the findings (Hindi & HR, 2020; Neria & Amit, 2004; Sumaji et al., 2019) that students experience difficulties in algebraic abstraction and only students who are gifted and brave do it. The same thing was also stated (Neria & Amit, 2004; Nizaruddin et al., 2020; Ulusoy & Incikabi, 2019) to be able to use algebraic language, students need to be accustomed to thinking models that are more different and more abstract than those accustomed to arithmetic, and students tends to backtrack on a solid base like a number or word.

Most students also have not been able to solve visual problems into verbal form, from the four information domains. Especially in terms of writing, in writing answers most students give answers directly without going through the stages of obtaining these answers so that researchers are not sure whether the answers written are fully done by these students. In TRM-02, the algebraic domain also proved to be the most difficult problem for students to solve. As Goldberg & Anderson (1989)

and Ceuppens et al. (2018) argued, even older students find it difficult to move between different components of a task (e.g., from the graphic to the text and back to the graphic) when solving a problem. As Goldberg & Anderson (1989) and Ceuppens et al. (2018) pointed out, most students find it difficult to change representations (from visual to verbal representations or vice versa).

Students' difficulties in solving algebraic representation problems occur because students do not have algebraic conceptual knowledge. This is in line with the opinion of Panasuk & Beyranevand (2010) and Nurrahmawati et al. (2021) that one of the indicators in solving problems is the ability to understand problems by recognizing the structural relationships shown through several different representations.

The research results that have been presented in general indicate that most students still experience difficulties in solving representation problems. This is in line with Lowrie & Diezmann (2009) research that "The ability to decode information graphics is fundamental to numeracy. However, the results of this study revealed that many students have difficulty decoding the graphics used in each of the graphical languages and that some languages are more difficult for students than others. The ability to decipher visual information is fundamental to arithmetic. However, the results of this study indicate that many students have difficulty translating the visual information used in each graphic information and there are domains that are more difficult for students to complete. When designing information into verbal representations, students must know sources of information such as text, keys, axes, and labels (Kosslyn, 2006). Therefore, it is necessary to consider all the components related to mathematics.

Another general observation is that of all ability levels, it turns out that each has questions that cannot be answered correctly. The level of difficulty of students mostly depends on the type of visual information itself. Baker et al. (2001) show with great confidence that although visual information is basically the same, each type of visual information has a major influence on students' successful completion. This was also reinforced by Lowrie et al. (2011) "*found that when students encountered mathematics tasks with high visual or spatial demands, realistic scenarios caused confusion if students were not willing (or able) to internalize all the information presented. Incorrect responses occurred when the student's personal experiences disrupted the problem-solving process*". Lowrie et al. (2011) found that when students are faced with math tasks with high visual or spatial information, it will cause student confusion if students are not able to internalize all the information presented. Errors can depend on the experience of students in solving problems. Therefore, if students are not able to create and interpret information effectively, it will affect the solutions given.

The interesting thing that the researcher was able to reveal was that the percentage of students' abilities in solving interpretation problems was higher than students' abilities in solving representation problems. Students are better able to solve problems based on the visual information presented compared to when students are asked to draw them.

CONCLUSION

Based on the results of the research and discussion conducted, it shows that there are still many mistakes and misunderstandings by students in completing each domain. In this case, it can be identified the types of problems that have the most influence on the successful completion of students. Based on these representation problems, some of them are the same but some are different for each level. According to the confession in the interview why this happened, the answers varied, namely:

The subject is too rushed or there is no connection between questions and conceptual understanding, declarative knowledge, procedural knowledge, and conditional knowledge. Memorizing problems are still needed in mathematics, so that the usual forgetting factor is very influential in responding to a question/test, or at least it can inhibit solving actions. Misconceptions occur (misunderstandings) so that what has been wrong so far is considered the truth..

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