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Learners' Graphical Efficacy When Solving Trigonometric Problems

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Abstract

This study explored grade 12 learners' graphical efficacy when solving problems involving trigonometric graphs. A structured test consisting of five trigonometric problems, with variations in context and structure, was administered to a purposefully selected group of 25 Grade 12 learners from the Sekhukhune District in South Africa. Insights into learners' graphing efficacy were obtained through task-based interviews. Data were analysed using direct interpretation which involved deductive thematic analysis of the task-based interviews and content analysis of the test scripts to match learners' responses to the themes drawn from the Meta-Representational Competence (MRC) framework. The results showed that most learners lack invention and functioning, critiquing and reflection efficacies and hence this affected their drawing and interpretation of the graphs and consequently lead to incorrect solutions. Furthermore, the results show most learners have critiquing efficacy. This indicates that learners lack graphical efficacy for solving trigonometric problems involving trigonometric functions. This finding has pedagogic implications: the apparent lack of graphical efficacy in graphical solutions may suggest inadequate mastery of the concept. Therefore, this study recommends that the teaching and learning of trigonometric graphs should consider the development of invention, functioning, critiquing and reflection efficacies.

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INTRODUCTION

Learners participating in scientific inquiry are required to present and analyze data as a means of problem-solving or as an end in itself (Scalise & Clarke-Midura, 2018). The creation and comprehension of graphics are crucial abilities that are part of a learner's scientific literacy. Lack of these efficacies restricts their application to comprehending and solving problems. Extensive research on basic graphical construction and interpretation revealed that learners have difficulties in making correct graph choices, allotting variables, labelling and scaling axes (Angra and Gardner, 2017). Most learners possess vast knowledge of representing algebraic functions. However, they find graphical construction and interpretation as complex and challenging activities (Glazer, 2011). Lowrie, Diezmann and Logan (2011) noted that learners' graphical construction and interpretation efficacies are affected by several variables such as graph characteristics, the content of the graph and learners' prior knowledge. Graphical representations are one form of mathematical representation.

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A representation refers to a structure or reference object in which a particular mathematics concept is expressed (Hill & Sharma, 2015). Mathematical concepts are easier to understand if they are represented in a visual form (Gur, 2009). Such graphical representations convey information during teaching and learning (Hebert & Powell, 2016). However, Zhang et al. (2021) regard representation as the critical feature of the spatial of creating a graph or model that enables abstraction to occur. Abstraction then takes place when a learner makes sense of the mathematical concept depicted by the graph. Trigonometry requires learners to link numerical and graphical relationships during problem-solving. The Cambridge examination report revealed that learners could not link them to underlying concepts. Learners find the graphs confusing and inconsistent with their conceptions (Cambridge Examination Report, 2019). In addition, Bornstein (2020) and Nejad (2016) revealed that learners struggle with the transformation of trigonometric functions.

Teachers also compound this problem by developing and drawing graphs without considering the 'how', 'what', 'where' or 'why' features of the graph (Adjei, 2018). However, in the context of South Africa, Makwakwa (2012) and the Examination Diagnostic Report of 2022 found that learners had many problems and misconceptions involving the construction and interpretation of graphs. We, therefore selected Sekhukhune District of Limpopo Province to study the problem in detail as it is one of the underperforming Districts in Mathematics in the Province (DBE, 2022; Nkadimeng, 2022). Learners disregard the mathematical ideas embedded in a graph and learn graphing rules by rote, rather than understanding their meanings and relationships with the broader mathematical ecosystem.

There is increasing evidence that learners' graphical literacy is enhanced by creating and refining their representations rather than by being taught specific graph types (Bodén & Stenliden, 2019). A study by Matuk et al. (2019) advocates that time must be spent reviewing and analysing learners' representations, and the representations of other learners, to reach a consensus that resolves toward the more standard graphs. Solving trigonometric problems sometimes requires the construction and interpretation of graphs. Graphs promote learners' ability to solve mathematical problems (Parrot & Leong, 2018). They also serve as tools for outlining relationships between variables. A graphical representation of a mathematical concept combines related information and supports comprehension (Matheson & Hutchinson, 2014). Cognitive functions of communication, knowledge construction, creative reasoning, and representation and enhanced engagement in learning are exercised in solving trigonometric problems using graphs (Batiibwe, 2020).

Görg et al. (2007) added that the use of graphs makes computations easier to perform compared to sentential representations. The use of graphs reduces the cognitive load on the problem solver and helps the limited ability of the mind to keep track of complex systems. The need for creating problem-solving graphs presents the problem solver with external cognitive aids (Jenlink, 2019). External cognitive aids such as graphs help the problem solver to reduce the amount of abstraction and reduce cognitive load. However, many learners lack the skills of decoding information from graphs to solve problems (Arsaythamby & Julinamary, 2015). Many researchers identified specific problems relating to learners' use of

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graphs as failure to draw appropriate inferences from a graph (Boote, 2014) and poor choice of graphs to use and drawing small graphs (Maries, Lin & Singh, 2017). Learning from a graph facilitates learners' understanding since visually presented information makes concepts more explicit and requires less inferential recognition than sentential representations (De Vries & Lowe, 2010). A study by Glazer (2011) indicates that learners do not spontaneously use graphs despite teachers using them during teaching and learning.

The majority of classes ought to promote the usage of graphs as efficient teaching and learning aids for mathematics (Darling-Hammond et al., 2019). Using a graph to solve math word problems is a good alternative approach (Uesaka et al., 2017). According to Quillin and Thomas (2015), learners should concentrate on graphs as a representational format. This study focuses on the efficiency of employing diagrams as a substitute way to help learners solve trigonometric problems. Abstract mathematical ideas are challenging to understand without a clear illustration. According to Slutsky (2014), a graph is a drawing that represents numerical data. Graphs aid in comprehension and problem-solving but are ineffective in the classroom (Davis & Arend, 2012). Information that is presented graphically might occasionally cause confusion in learners, according to Schneider et al. (2010). Learners also have trouble interpreting graphs, which negatively affects their ability to solve problems. According to Murata (2008), learners should become knowledgeable about a variety of graphs in order to develop experience and a strong foundation. Similarly, Maries and Singh (2018) found that learners' ability to construct graphs correlates with successful problem-solving. They corroborated the positive correlation between a learner's approach to a problem and representations found by Koedinger and Nathan (2004). Certain representations predict the use of a particular solution strategy. Graphs make unseen mathematical concepts visible (Arcavi, 2003). Graphical representations are superior to symbolic representations when solving problems because they provide a visual representation of the concept (Maries & Singh, 2016). Drawing a graph is a recommended approach for representing a word problem, mainly as learners work towards higher levels of mathematics in middle or high school grades (Poch et al., 2015). According to Hindi and HR (2023), graphs are one of the most commonly used mathematical functions to represent information in textbooks, standardized tests, and other printed and electronic media. In mathematics, graphs serve as external visual aids that facilitate problem cognition. Graphs also serve as thinking and communication tools without which memory, thought, and reasoning are all constrained (Meirelles, 2013). Furthermore, a graph is a semiotic representation that stands for something else (van Garderen et al., 2014). A graph represents concreteness and connects concepts with their referents (Fyfe et al., 2014).

Learners’ graphical efficacy is an important requirement for solving trigonometric function problems (Mohamed et al., 2023). Graphical efficacy refers to the skills or abilities and efficiency of learners in constructing graphs as well as being able to interpret their features. It is a concept often used to describe how well learners use visual elements to effectively solve mathematical problems. In studying trigonometric functions, learners are expected to be able to draw, use and interpret graphs during problem-solving (Halim, et al., 2023). According to Wulandari et al. (2020), learners exhibit graphical efficacy if they are able to

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convert mathematical information into graphical functions. This study draws from and concurs with Yusrina et al. (2020) who pointed out that learners exhibit graphical efficacy when they are able to draw the graph from the given equation and are able to interpret a drawn graph. This shows that learners should be able to switch between different representations when solving function problems namely: visual which includes graphs, verbal, and algebraic or symbolic representations (Fudin et al., 2022).

Therefore, the purpose of this study was to explore how well Grade 12 learners could draw and analyze trigonometric graphs. The study explored learners' efficacies when drawing and interpreting are in constructing and interpreting graphs and discusses some of the challenges that learners experience while solving problems involving graphs.

Understanding today's world and being scientifically educated requires graphic competency (Glazer, 2011). The ability to create and analyze trigonometric graphs is required of learners. The relevance of graphing cannot be overstated; most curricula need learners to utilize graphs to answer questions in mathematics and the sciences. Graphs are effective teaching and thinking aids for mathematics (Carter, 2010). A mathematical concept's graphic depiction can be used as a thinking and communication tool. Meirelles (2007) emphasized that using a variety of strategies and choosing the right representation are essential components of problem-solving. Graphs should consequently be used by learners as alternate tools for problem-solving (Stylianou, 2011). They ought to appreciate the use of graphs in problem-solving and use them as an alternative to other tools (Maries & Singh, 2018).

Graphical constructions and interpretations are prominent skills that high school learners must possess when solving problems involving trigonometric graphs (Matuk et al, 2019). Graphs are iconic symbols, hence more concrete and more comfortable to understand and use in problem solving than abstract or theoretical equations. However, South African learners have challenges in solving problems involving trigonometric graphs (DBE 2021; 2022). The Department of Basic Education noted that grade 12 learners were unable to draw and interpret trigonometric graphs. The interpretation of trigonometric function graphs was identified as the main area where learners lack proficiency (Rosjanuardi & Jupri, 2020). Furthermore, they lack skills of solving problems involving trigonometric graphs that integrate other mathematical concepts (Herscovics, 2018). Therefore, there is a need to look into what skills for solving problems involving trigonometric graphs do learners possess. For learners to solve trigonometric graphical problems effectively, they must possess efficacies or skills of translating the information and conditions stated in the problem into a visual pictorial form and vice-versa. These efficacies include invention, functioning, critiquing and reflection (diSessa, 2004). However, there is a dearth of literature as to what efficacies for solving trigonometric graphs learners possess and how these affect their solution processes.

Therefore, this paper explores grade 12 learners' graphical efficacy when solving problems involving trigonometric graphs. Learners' graphical efficacy will assist teachers to emphasize on the acquisition of these efficacies during teaching and learning processes.

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learners' graphical efficacy and its effect on their problem-solving competencies. The study focused on the translation of algebraic trigonometric problems into a graphical or schematic representation and vice-versa. The following research questions guide this study:

a. What graphical efficacies do learners use to construct and interpret trigonometric graphs?
b. How do learners' graphical efficacies affect their solution of trigonometric function problems?

THEORETICAL FRAMEWORK

Meta-Representational Competence (MRC) by diSessa (2004) guides this study. The MRC framework has its roots in Cognitive Psychology; it explains how learners construct and interpret scientific representations. MRC is a set of critical abilities and understandings about how and why learners prefer to use specific mathematical representations. It models learners' competence and ability to produce and use external representation. MRC describes learners' competence with producing and interpreting particular representations. It also involves the use of invented or existing representations, functions they might serve and their relative applicability and efficacy in different contexts. Thus, MRC transcends the understanding of the function and operation of a specific scientific representational system (diSessa & Sherin, 2000). The Meta-representational Competence (MRC) framework describes learners' conceptual understanding of information; learners' communication of their conceptual understanding to others; and the evaluation of learners’ conceptual understanding of information. The MRC framework consists of four components which describe learners' abilities as they work on a mathematical problem; invention, critique, functioning, learning or reflection. These four constructs underlie skills and abilities that allow learners to conceive different representations. The invention refers to the learner's skills and abilities to conceive different graphical representations. It involves the classification of variables, deciding on data manipulation and finalising the graph choice. The learner is expected to actively interpret the graph with appropriate mechanics. In the critique phase, the learner is expected to exhibit critical knowledge that is essential for assessing various types of graphs and their strengths and weaknesses. In the functioning phase, the learner is required to reason and show knowledge of different representations and their purposes in different problem contexts, their usage and limitations.

Thus, the functioning phase unearths learners' reasoning for understanding the purpose of different types of graphs, and their usage is dependent on the type of data present. The reflection phase consists of critical reflections on graph alignment, graph choice and data representation. This step is similar to Polya's "look back" stage, where the interest is to develop learners' habits of developing systematic intellectual skills for solving problems. Thus, the phases reveal learners' awareness of their understanding of graphs and gaps in their knowledge.

Table 1. Components of the MRC framework Source di Sessa and Sherin (2000)

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Component of MRC	**Definition**	**Relevance to Graphing**
Invention	The underlying skills and abilities that allow learners to conceive novel representations	Competency with graph choice, construction, and knowledge of variables is vital for conjuring graphical
Critiquing	Critical knowledge that is essential for assessing the quality of representations	Assessing the strengths and weaknesses of various graphs expose learners' critical knowledge
Functioning	Providing reasoning for understanding the purpose of different representations, their usage, and limitations	Functioning unearths learners' reasoning for understanding the purpose of different types of graphs, and the usage is dependent on the type of data present.
Reflection	Strategies for fostering understanding of representations	Reflection reveals learners' awareness of their understanding of graphs and gaps in their knowledge

RESEARCH METHOD

This study employed a qualitative approach within an interpretive paradigm. Qualitative research is useful when investigators want to have a clear and deep understanding of the human condition that causes human behaviour (Bogdan & Biklen, 1982). Condition in this study is solving trigonometry problems whereas behavior is learners' graphical efficacy. Therefore, in this study, the meanings of learners' solutions to trigonometric problems were interpreted to establish their graphical efficacy (Rahman, 2017).

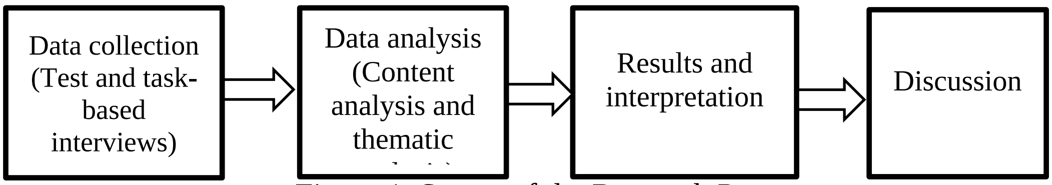

Figure 1**.** Stages of the Research Process

To realize the purpose of this study, an intrinsic case study design (Stake, 1995) of conveniently selected whole class of 25 Grade 12 learners attending mathematics at a high school in Sekhukhune District, Limpopo Province – South Africa was used. The sample consisted of 14 males and 11 females, their age ranges from 18 – 20 years. The sample was selected because of their accessibility to the researcher and trigonometric graphs is one of their prescribed topics in the curriculum (DBE, 2011) which grade 12 learners experience some challenges. An intrinsic case study was chosen, as the study was not intended for transferability purposes. The study followed the research stages outlined in figure 1. Data were collected using a test and task-based interviews during winter enrichment classes. The test that was developed by the first author was given after learners were taught how to solve trigonometry function problems using graphical and algebraic approaches in order to investigate their solution process. In order to ensure data source triangulation, task-based interviews were conducted. Task-based interviews were conducted to obtain rich data about learners' efficacies based on learners' responses to trigonometric graphs problems. Learners were purposively selected based on their responses in order to probe further into their shortcomings.

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All the collected data were analysed using directed (deductive) content analysis of the test scripts in order to identify the themes (Hsieh & Shannon, 2005, Zhang & Wildemuth, 2016) and direct interpretation which involved thematic analysis of the task-based interviews (Patton, 2014). For content analysis, the data was fitted or matched with the tenets of the framework, that is invention, critiquing, functioning and reflection. The researchers extracted content from texts (learners' responses) to examine meanings, themes and patterns. During the analysis of learners' responses to the test and interviews, the competencies envisaged were abilities to draw graphs, analyse and interpret graphs and extract the main ideas from graphs. The analysis focused on skills such as invention efficacies, efficacy to critique graphs, functioning efficacies and reflection efficacies. The themes were merged during the discussion of the results

RESULTS AND DISCUSSION

The written test questions assessed learners' graphical efficacies, capacities and skills of extracting information from graphs and use of graphs as problem-solving tools. Participants' responses are discussed, evaluated and measured against the meta-representation components.

Invention, Functioning and Reflection Efficacies

Item 1.3 required learners to draw a detailed graph of $-2tan\frac{3}{2}x$ for the interval $x \in [-120°; 180°]$. This item assessed the learners' analytical skills of transforming algebraic problem situations into a graphical representation, that is, their invention skills. The question requires learners to interpret the question and draw a representation of the situation. Drawing of graphs posed some challenges to most learners. Of the learners who attempted to draw the graphs, most of them did not construct them correctly. Some learners were unable to derive the correct scale which resulted in incorrect graphs. Some learners used incorrect scales, hence producing incorrect graphs. Their graphs were either had incorrect turning points, do not have asymptotes or not drawn within the given interval.

Incorrect scale and incorrect diagram

Most learners were unable to come up with the correct scale hence they drew the incorrect diagram. For item 1.3, these learners did not show the asymptotes and the intercepts with the axes on their graphs. This resulted in them drawing an incorrect graph. For example, learner C's graph in figure 2.

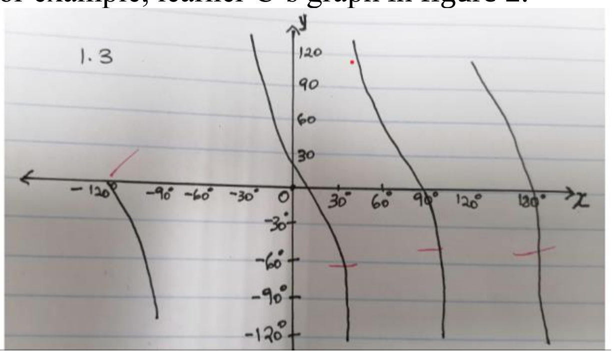

Figure 2. Learner **C**'s graph of $f(x) = -2tan\frac{3}{2}x$

During task-based interviews, learner C indicated that she "used this scale because in algebra, the values are normally the same in both axes". Hence, she used

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the same scale in both axes. This indicates that the learner is not aware of her gaps in knowledge, the learner lacks reflection efficacy. This clearly indicates that the learner drew the cartesian plane and put both the x- and y-values before coming up with the table of values or deriving the table from the calculator. The learner may have not been able to use a calculator. She thought that the Algebra strategy could be used in trigonometry. The learner is not aware of what should go into the x-axis and the y-axis when drawing trigonometric graphs.

Some learners made an incorrect choice of the graph. For example, in figure 3, learner M, besides using the incorrect scale, the learner also made an incorrect choice of a graph. She drew the tangent graph which has turning points.

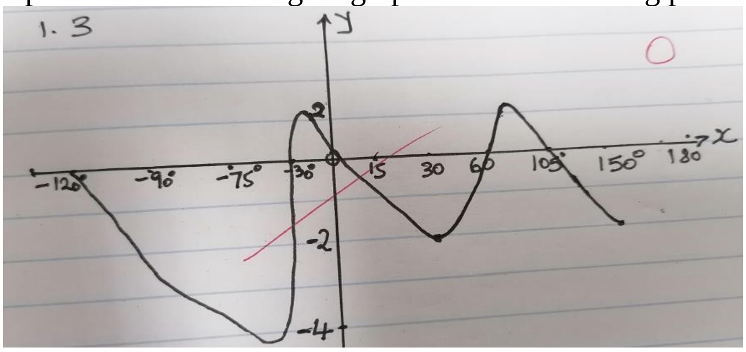

Figure 3. Learner **M**'s graph of $f(x) = -2tan\frac{3}{2}x$

During the interviews, this learner indicated that "I was confused. Initially, when I was punching on my calculator, I could see that the points were not joining each other. So, I decided to join them as I thought that I am making a mistake of not joining them". This shows that she did not conceive the tangent graph well and hence lacks functioning efficacy. Her conception of plotting the graphs is that every graph must have a turning point, hence she joined the points together so that at the end the graph has turning points. However, the learner reflected on her gaps in knowledge about her understanding of tangent graphs. The graphs drawn by most learners show that they lack graphic skills. The difficulty in constructing the graphs is inconsistent with di Sessa (2004) who indicated that graph construction (invention) is not as problematic as critiquing and reflecting on the graph itself. However, the results are consistent with Angra and Gardener (2018) who revealed that graph construction is more complex, and learners find difficulty in constructing or drawing correct graphs. Failure to draw the graph means that learners lacked invention competencies as they could not construct and lacked knowledge of selecting appropriate scales that will lead them to an appropriate graphical representation.

Correct scale but a partially correct diagram

Some learners used the correct scale and showed the asymptotes by the solid lines as well as drawing the reflection of the given graph on some parts of their diagram for item 1.3. For example, in figure 4, for learner P, the middle part of the graph which cuts the x-axis at zero is

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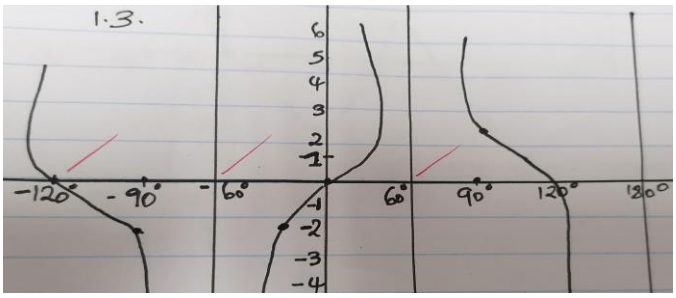

Figure 4. Learner **P**'s graph of $f(x) = -2tan\frac{3}{2}x$

The reflection of the given graph along the y-axis. This learner did not take the given interval into consideration and hence lacks the functioning efficacy. He also drew the graph in the interval below $-120°$. It could be that for him the tangent graph should appear in both the positive and the negative y-axis. The learner did also not take into consideration that the graph should move closer to the asymptotes. During the interviews, this learner indicated that "the tangent ratio is always positive in the first quadrant, hence the graph is positive between $0°$ and $60°$. Negative angles are in the fourth quadrant, hence when $\theta = -30°$, the graph appears in the fourth quadrant". This learner lacks functioning and reflection efficacies as he failed to conceive that the degrees on the x-axis are the ones telling the quadrants, not the cartesian plane.

Some learners used the correct scale and the correct asymptotes, however, drawn the graph in an incorrect interval. For example, in figure 5, learner E realised that the graph should not appear in both the positive and the negative y-axis at $-120°$. The learner lacked functioning efficacy as she did not realise that in the positive y-axis, the x-values will be less than -120° hence outside the given interval.

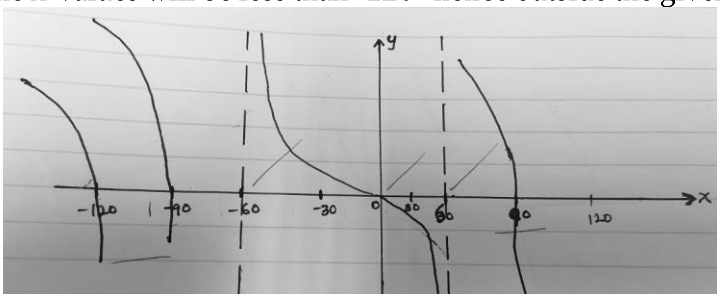

Figure 5. Learner **E**'s graph of $f(x) = -2tan\frac{3}{2}x$

This learner indicated that he was a bit confused about where to draw the last part of $-120°$ because he was not sure if it should be up or down. He further said: "I was not sure if $-120°$ and $120°$ should be the asymptotes or not". The findings in figures 4 and 5 These findings indicate that learners partially possess invention efficacy as they were able to use a correct scale however, drew partially correct diagrams. These findings are inconsistent with Angra and Gardener (2018) who found that learners had difficulties in identifying the correct scale as well as labelling and scaling the axes.

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Correct scale and Correct Graph

Some learners managed to use a correct scale and drew the correct asymptotes and correct graph. For example, learner I in figure 6 indicated the invention, functioning and reflection efficacies.

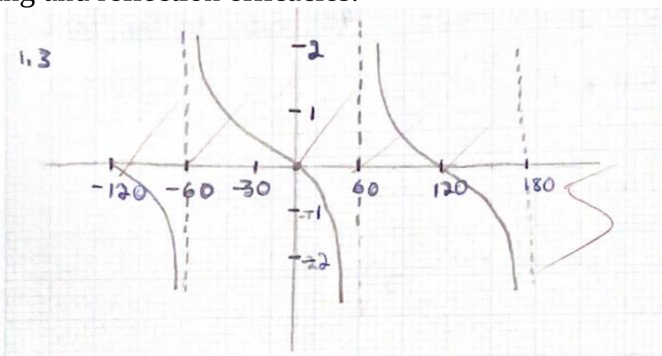

Figure 6. Learner **I**'s graph of $f(x) = -2tan\frac{3}{2}x$

This study's findings reveal that some learners were able to use a correct scale and drew the correct diagram. These results show that these learners exhibit all the graphical efficacies according to Wulandari et al. (2020) because they are able to convert mathematical information into graphical functions. Furthermore, these results are inconsistent with Angra and Gardener (2018) who found that learners find difficulty in graph construction and identifying the correct scale, labelling and scaling the axes.

Critiquing, functioning and Reflection Efficacies

For item 1.5 learners were expected to describe the transformation of the graph of f(x) to form the graph of the function: $g(x) = -2\tan(\frac{3}{2}x + 60°)$. This item assessed the learners' analytical skills of transforming graphical or schematic representation problem situations into a algebraic form. Learners needed to interpret the visual representation format into algebraic forms. Learners were required to solve the problem from the resulting visual representation. They were expected to interpret the graph they have drawn in item 1.3.

Description of the transformation

For item 1.5 one of the learners indicated that the graph shifted 60 °to the left. Learner L, in figure 7, was able to describe the transformation correctly. This indicates that this learner understood the functioning of the shifting of graphs.

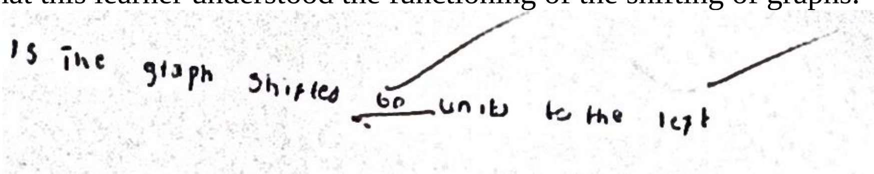

Figure 7. Learner **L**'s Description of the graph of $g(x) = -2\tan(\frac{3}{2}x + 60°)$.

This learner indicated when interviewed that 60 units were taken from the equation and the addition sign describes the shifting of the original graph to the left side. This shows that the learner has functioning and reflection efficacies. This finding is inconsistent with Bornstein (2020) who found that learners had challenges in describing the transformations of trigonometric functions.

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The correct side of shifting but incorrect units

Most learners indicated that the graph shift 40 units to the left. For example, in figure 8, learner P indicated that the graph is shifting 40 units to the left. Though this learner was able to understand that the graph will shift to the left, they did not realise that it is shifting by 60 units. This learner factorised the $\frac{3}{2}x + 60°$ to be $\frac{3}{2}(x + 40°)$, then disregarded the $\frac{3}{2}$ and concentrated on the $(x + 40°)$ only.

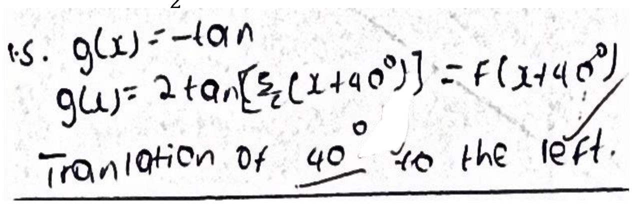

Figure 8. Learner **P**'s Description of the graph of $g(x) = -2\tan\,(\frac{3}{2}x + 60°)$.

During the interviews the learner indicated that he focused on $(x + 40°)$ only after factorisation. Hence, he thought that the shifting units are 40 instead of 60. Though the learner portrays critiquing efficacy by being able to factorise the equation, the learner lacked functioning efficacy with regard to graph interpretation.

Incorrect description of the transformation

Some learners describe the translation as a shift by 40 units to the right. These learners' critiquing of the graph is wrong. For example, Learner F describes the transformation as shown in figure 9:

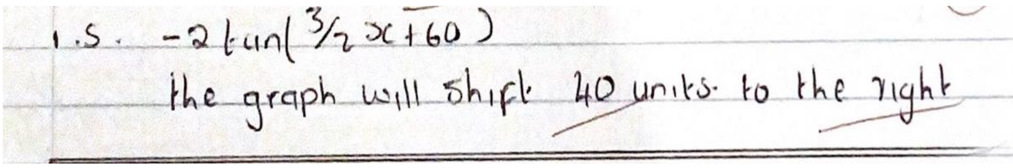

Figure 9. Learner **F**'s Description of the graph of $g(x) = -2\tan\,(\frac{3}{2}x + 60°)$.

Learner F said that he started by factorising the equation first. He then got $(x + 40°)$ in brackets which means that it is 40° to the right because of the addition sign. This means that the learner lacked reflection efficacy. These findings in Figures 8 and 9 are consistent with Nejad (2016) who found that learners experience difficulties with describing transformations of trigonometric graphs.

The second question assessed the learners' skills of interpreting graphical representations of trigonometric functions. Learners were expected to demonstrate their critiquing, functioning and reflection efficacies of the meta-representation competencies. Participants were expected to interpret graphical information and use it to respond to a set of questions. Firstly, they were required to use the graph to find the equations of the graphs and later use the graphs to solve the equations. For item 2.1, learners were expected to determine the values of a, b, c, d, p, q, r and s and hence the equation of $f(x) = a\ sin\ (bx + c°) + d$ ***and*** $g(x) = p\ cos\ (qx + r°) + s$. The main idea in the item was to interpret the graphs in order to determine the parameters. Item 2.2 required learners to read from the sketch, the values of x for which $f(x) = g(x)$for $x \in [0°; 180°]$. Item 2.3 required learners to find the values of x for which $f(x) \geq g(x)$. Most learners did not attempt these items. This

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indicates that the learners had knowledge gaps of graphical interpretations and the use of variables in conjuring new graphical representations such as changes in amplitude and vertical and horizontal shifts of the graphs due to change in parameters. Thus, learners lack invention, critiquing and functioning skills therefore dysfunctional with regard to graphing efficacy.

Incorrect Identification of Parameters

Figure 10 shows that learner Q, wrote all the values of the variables for item 2.1 from the y- axis. It could be that the learner was just copying the values on the y-axis on the graph for each variable given. This indicates that the learner lacked invention, critiquing, functioning and reflection efficacies.

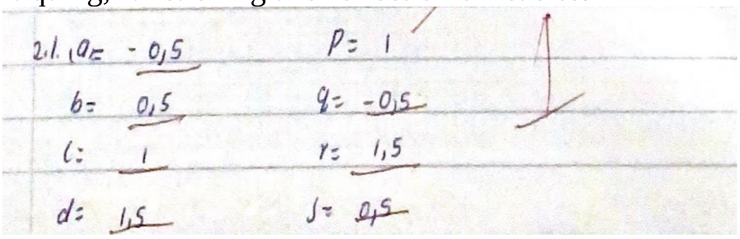

Figure 10. Learner **Q**'s determination of the Parameters of $f(x) = a\,sin\,(bx + c°) + d$ ***and*** $g(x) = p\,cos\,(qx + r°) + s$.

The learner did not know where to read each parameter from the graph. Hence, she did not even consider the degrees guideline on the r parameter. This shows that the learner lacked both critiquing and reflection efficacies as she was unable to decode information from the graph. These results are in line with Arsaythamby and Julinamary (2015) who indicated that many learners lack the skills of decoding information from graphs.

Reading off values from the Graph

Most learners who attempted item 2.2, got a value of $90°$ which was correct, however they did not get other values correct. This indicates they managed to read it from the graph, hence they possess the functioning and reflection efficacies. However, learners struggled to estimate or read other values from the graph. This indicate that learners lacked critiquing and reflection efficacies. For example, learner L got $90°$ correct but struggled to get the other variables.

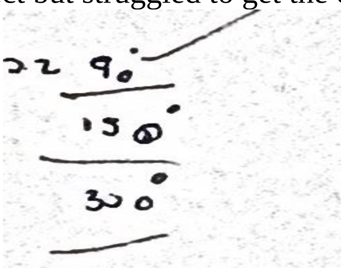

Figure 11. Learner **L**'s determination of Points of Intersections $f(x) = a\,sin\,(bx + c°) + d$ ***and*** $g(x) = p\,cos\,(qx + r°) + s$.

In figure 11 learner L was able to realise that the two graphs intersect at $90°$ and the other points are less than $150°$ and less than $300°$. She exhibited the lack of critiquing the graphs. Learner L indicated during the interviews that the point

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between $120°$ and $150°$ is $130°$. This indicates that she did not interpret the scale correctly as such she did not estimate the value to be $145°$. This shows that the learner lacks functioning and reflection efficacies as she was unable to read off and estimate the values of unknown variables from the graphs; hence she was unable to solve equations and inequalities. This is consistent with the findings of Herscovics (2018), who found that learners' lacked knowledge of some of the mathematical concepts embedded in the given graph. This indicates that learners lack reflection efficacy as they could not link mathematical concepts.

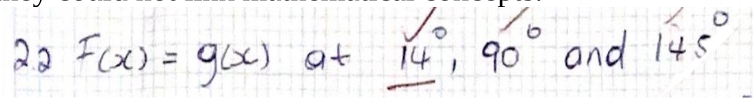

Figure 12. Learner **B**'s determination of Points of Intersections $f(x) = a\ sin\ (bx + c°) + d$ ***and*** $g(x) = p\ cos\ (qx + r°) + s$.

Learners showed an understanding of the use of graphs to solve trigonometric inequalities by reading solutions from the points of intersections of the graphs in item 2.2. Learner B's response to 2.2 affected her response to 2.3.

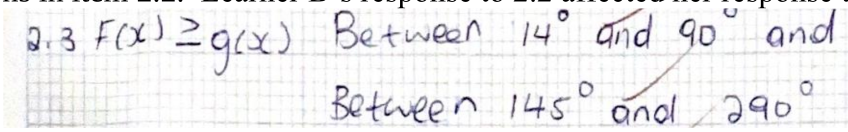

Figure 13. Learner **B**'s determination of Points of Intersections $f(x) = a\ sin\ (bx + c°) + d$ ***and*** $g(x) = p\ cos\ (qx + r°) + s$.

During the interviews, learner B indicated that she used the x-values she got in item 2.2. to answer item 2.3 and got the answer she wrote in figure 13. This indicates the learner possess reflection and functioning graphical efficacies as she managed to interpret the graph by switching from trigonometric equations to trigonometric inequalities using the same graph. This finding is in line with Yusrina et al. (2020) who indicated that learners should be able to draw and interpret the drawn graphs

CONCLUSION

This paper explored learners' graphical efficacy when solving problems involving trigonometric graphs. The analysis of the results reveals that most learners have limited graphical efficacy, and this affected their solution processes. Most learners lack invention skills as evidenced by the use of incorrect scales hence drawing incorrect graphs. However, others used a correct scale but produced partially incorrect graphs. Most learners showed a lack of functioning, critiquing and reflection efficacies with regard to graphical interpretation. They struggled to read off values of the parameters from their own drawn graphs and the given drawn graph. Furthermore, the results of this study showed that most learners lacked reflection as well as critiquing as they were unable to describe the transformation of functions. The study found that participants struggled with switching between the graphical and algebraic representations of trigonometric graphs and equations. It has been discovered from the results of this study that the meta-representation efficacies are interwoven such that learners who lack one become limited in their proceeding to the other, however, learners may possess partial invention but lack all other efficacies. Therefore, this study recommends that the development of all these graphical efficacies be stressed during trigonometry teaching. The systematic

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teaching of trigonometric graphs and equations with multiple and flexible representations needs to be a priority in high school mathematics classrooms. Emphasis must be on skills of creating and extracting information from graphs to solve problems. Teaching should not focus on technical skills such as drawing of graphs only but rather on the relationships between the different representations. Steenpaß and Steinbring (2014) also emphasised that during the teaching of graphs, the graph's single elements have to be correlated with each other and interpreted as parts of a complex symbol system to become meaningful.

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