

Integrating the Case Studies Pedagogy and the Anthropological Theory of Didactics to Enhance the Teaching and Learning of Algebra in Undergraduate STEM Education

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Abstract

Methodology case studies plays a pivotal role in STEM education, especially in teaching undergraduate algebra. This article emphasizes the power of real-world scenarios and active pedagogy, a student-centered approach that makes abstract algebraic ideas tangible. The Anthropological Theory of Didactics (ATD) is spotlighted as a perspective that treats mathematical knowledge as a cultural artifact, embedding mathematics concepts deeply within cultural and historical contexts. By comparing active pedagogy with ATD, the article unveils a compelling overlap that values the significance of context in learning. Students don't just engage actively but also delve into the cultural essence of mathematical problems, gaining a profound understanding. Addressing the hurdles undergraduates face in transitioning from high school algebra to university level, the article suggests activities that merge active pedagogy and ATD. This holistic approach has the potential to transform algebra education into a more engaging, relevant, and culturally resonant experience.

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INTRODUCTION

In the rapidly evolving landscape of Science, Technology, Engineering, and Mathematics (STEM), the application of methodology case studies has emerged as an invaluable tool for in-depth exploration and understanding. These case studies provide a focused lens through which specific instances or scenarios within the STEM disciplines can be scrutinized. By delving into the intricacies of methodology application, its challenges, and outcomes, these studies shed light on the nuances of practical implementation in STEM research and education.

The Anthropological Theory of Didactics (ATD) offers a unique lens through which to examine the teaching and learning of mathematics. Rooted in anthropological perspectives, this theory seeks to understand mathematical knowledge as a form of cultural knowledge, emphasizing the practices, norms, and values that shape mathematical activities in various contexts (Chevallard, 2006). The world of education, particularly in the realm of mathematics, has been witnessing a convergence of theories and methodologies. Two predominant themes

in this convergence are the Anthropological Theory of Didactics (ATD) and the surge in active pedagogy case studies. These two domains, although originating from different paradigms, intersect in their pursuit of enhancing mathematical understanding and fostering meaningful learning experiences.

The use of case studies active pedagogy for the teaching and learning of algebra at the undergraduate level offers a refreshing departure from traditional methodologies. By situating algebraic concepts within real-world contexts and emphasizing collaborative problem-solving, this approach fosters a deeper, more integrated understanding of algebra. As educators continue to seek innovative strategies to enhance mathematical instruction, the case studies active pedagogy emerges as a promising contender, bridging the gap between theory and practice in the realm of algebra.

It is noteworthy that the article culminates in suggesting activities that meld active pedagogy with ATD tenets to amplify the experience of learning algebra. This in-depth examination highlights the transformative power of modern teaching methodologies, making algebra instruction more immersive, pertinent, and deeply rooted in cultural significance.

Objectives

The pursued objectives are summarized as follows:

1. **To Understand the Significance of Methodology Case Studies in STEM:** The objective is to shed light on the role and importance of methodology case studies in the evolving landscape of STEM disciplines.
2. **To Explore the Anthropological Theory of Didactics (ATD) in Mathematics Education:** This article seeks to delve into the ATD's anthropological perspectives on teaching and learning mathematics. By emphasizing the cultural, historical, and social contexts of mathematical concepts, the objective is to highlight the theory's contribution to a more holistic and integrated understanding of algebra.
3. **To Highlight the Benefits of Active Pedagogy in Algebra Education:** Through this objective, this article aims to promote the use of active pedagogy in the teaching and learning of algebra at the undergraduate level. By situating algebraic concepts within real-world contexts and advocating for collaborative learning experiences, this article underse
4. **To Address the Challenges Faced by Undergraduate Students in Algebra:** The final objective is to provide a comprehensive exploration of the difficulties students encounter when studying algebra at the undergraduate level. By identifying common challenges, we provide some proposals of activities about how to introduce the Case Studies and the ATD for the learning and teaching of Algebra at undergraduate levels in STEM titulations.

Theoretical Background

Methodology case studies in STEM are characterized by their detailed investigations into the processes, techniques, and tools used in research or teaching. Unlike broad surveys or experimental designs, they concentrate on the specifics of a method, offering insights into its applicability, limitations, and potential enhancements. One of the inherent advantages of this approach is the opportunity for rich data collection. Drawing from multiple sources such as observations, interviews, and document reviews, these studies can paint a comprehensive picture of the method in question (Merriam, 1998).

Furthermore, the real-world scenarios presented in these case studies play a pivotal role in bridging the gap between theoretical concepts and their practical applications in STEM. Students and professionals alike benefit from these tangible examples, which elucidate abstract principles and demonstrate their relevance in real-world contexts (Loucks-Horsley et al., 2003). The qualitative nature of these case studies also offers a degree of flexibility, allowing researchers to adapt and refine their approach as they delve deeper into the subject matter. This adaptability ensures that the insights derived are both relevant and timely, reflecting the dynamic nature of STEM disciplines.

As STEM fields continue to push the boundaries of knowledge and innovation, the role of methodology case studies becomes ever more crucial. By offering a detailed, practical perspective on specific methods and techniques, these studies contribute significantly to the advancement of research, teaching, and learning in STEM.

In the undergraduate setting, the application of ATD to mathematics education provides valuable insights into the ways in which mathematical concepts are introduced, developed, and institutionalized. By considering mathematics as a cultural endeavor, ATD shifts the focus from mere content acquisition to a deeper understanding of the cultural and historical roots of mathematical ideas, as well as the social practices that surround them.

One of the central tenets of ATD is the notion of "praxeologies". A praxeology refers to a specific type of mathematical activity, encompassing both the practical tasks and techniques associated with that activity and the theoretical justifications that underpin it (Barbé, Bosch, Espinoza & Gascón, 2005). Within the undergraduate mathematics classroom, praxeologies come to life as students grapple with mathematical problems, develop and apply techniques to solve them, and delve into the theoretical frameworks that justify these techniques.

Furthermore, ATD emphasizes the dynamic nature of mathematical knowledge. As undergraduates transition from high school to tertiary education, they encounter shifts in the mathematical praxeologies they engage with. These

shifts are not merely content-based but also involve changes in the ways mathematical tasks are approached, the techniques deemed acceptable, and the theoretical discourses that are privileged (Winsløw, 2003).

By applying ATD to undergraduate mathematics education, educators are better positioned to recognize and address the challenges students face in this transitional phase. Understanding the cultural shifts in mathematical praxeologies can help educators design curricula and teaching strategies that bridge the gap between high school and tertiary mathematics, fostering a deeper and more integrated understanding of mathematical concepts.

Moreover, ATD promotes a holistic view of mathematics, acknowledging its historical and cultural roots. By situating mathematical concepts within broader socio-cultural contexts, undergraduate students are encouraged to view mathematics not as an isolated discipline but as an integral part of human culture and history.

Active pedagogy emphasizes student-centered approaches, wherein learners actively participate in their education, often through collaborative and experiential activities (Bonwell & Eison, 1991). Case studies within this domain highlight the diverse strategies educators employ to shift from traditional lecture-based formats to more interactive and engaged learning environments. Whether it's through problem-based learning, group discussions, or hands-on projects, active pedagogy champions the idea that students learn best when they are active participants in the learning process (Prince, 2004).

On the other hand, the Anthropological Theory of Didactics (ATD) posits mathematical knowledge as a cultural artifact, deeply embedded in human practices and histories. Central to ATD is the concept of "praxeologies" - combinations of practical tasks and their associated techniques, along with the theoretical justifications underpinning them (Chevallard, 2006). In essence, ATD seeks to understand the cultural and praxeological dimensions of mathematical activity, emphasizing the sociocultural contexts in which mathematical knowledge is developed and transmitted.

When juxtaposed, active pedagogy case studies and ATD present an intriguing overlap. Both paradigms recognize the importance of context in learning. While active pedagogy emphasizes experiential and contextual learning experiences, ATD situates mathematical knowledge within its broader cultural and praxeological contexts. This overlap suggests that active pedagogical strategies, when informed by ATD's anthropological lens, can provide students with a richer and more culturally grounded understanding of mathematical concepts (Ruthven, 2011).

Several case studies have highlighted the benefits of integrating ATD perspectives into active pedagogical strategies. For instance, when students engage in collaborative problem-solving tasks, not only are they actively participating in their learning, but they are also delving into the cultural and praxeological dimensions of the mathematical problems they are tackling (Gueudet & Trouche, 2009). This integrated approach allows students to see mathematics not just as abstract concepts, but as cultural tools with real-world implications.

Furthermore, by recognizing the cultural roots of mathematical knowledge, educators can design active learning experiences that are more inclusive and culturally responsive. Active pedagogy case studies have shown that when mathematical tasks are grounded in students' cultural contexts, they resonate more deeply, leading to enhanced understanding and retention (Aguirre, Mayfield-Ingram, & Martin, 2013).

Hence, the convergence of active pedagogy case studies and the Anthropological Theory of Didactics offers a promising avenue for enhancing mathematics education. By integrating active, student-centered approaches with a deep anthropological understanding of mathematics, educators can foster learning experiences that are both engaging and culturally meaningful. *The Use of Case Studies Active Pedagogy for the Teaching and Learning of Algebra at the Undergraduate Level*

Algebra, a foundational component of undergraduate mathematics, often presents challenges for both educators and learners. Traditional teaching methods, which rely heavily on lectures and rote memorization, may not effectively convey the dynamic nature of algebraic thinking. Enter the realm of case studies active pedagogy, a transformative approach that has shown promise in enhancing the teaching and learning experience in algebra courses at the undergraduate level.

Case studies active pedagogy is characterized by its emphasis on student-centered learning, where real-world scenarios and problems serve as the foundation for exploration and discovery (Bonwell & Eison, 1991). Instead of merely presenting algebraic concepts in abstraction, this approach situates them within tangible contexts, allowing students to grasp the practical applications and implications of algebra in various fields, from engineering to economics.

One of the defining features of this pedagogical approach is its emphasis on collaborative problem-solving. Students work in teams, delving into algebraic problems presented within the case studies. These problems are designed to be open-ended, prompting students to discuss, debate, and collectively arrive at solutions (Prince, 2004). Such collaborative endeavors not only enhance understanding but also foster critical thinking and communication skills, essential competencies for budding mathematicians and professionals.

Furthermore, the case studies approach taps into students' innate curiosity. By presenting real-world challenges that require algebraic solutions, educators can ignite students' interest and motivation. For instance, a case study might explore the dynamics of population growth using algebraic functions or delve into the complexities of financial markets through polynomial equations. These real-world connections make algebra relatable, demystifying its complexities and showcasing its relevance (Laursen, Hassi, Kogan, & Weston, 2014).

The integration of technology further enhances the efficacy of case studies active pedagogy. Digital tools and platforms can facilitate dynamic simulations, allowing students to manipulate algebraic variables and witness the outcomes in real-time. Such interactive explorations provide immediate feedback, enabling students to experiment, iterate, and refine their understanding (Borba & Villarreal, 2005).

Challenges faced by students in Algebra at undergraduate level: A short exploration

Algebra, a cornerstone of mathematical study, presents a myriad of challenges for undergraduate students transitioning from high school to higher education. This transition often amplifies the complexities of algebraic concepts, revealing gaps in foundational knowledge and introducing new, more abstract ideas. Understanding the challenges faced by students in undergraduate algebra is crucial for educators aiming to provide effective instruction and support.

1. Conceptual Understanding

One of the most significant challenges students encounter is moving from a procedural understanding of algebra, often emphasized in earlier education, to a conceptual one. While students might be adept at rote manipulation of symbols and equations, they often struggle to grasp the underlying concepts these symbols represent (Hiebert & Carpenter, 1992). This lack of deep understanding can hinder their ability to apply algebraic knowledge in novel contexts or to more advanced topics.

2. Abstract Reasoning

Algebra at the undergraduate level introduces students to a higher degree of abstraction. Concepts like abstract algebraic structures, fields, and vector spaces can be particularly daunting as they often lack the tangible representations students might have encountered in earlier algebra courses (Dubinsky, 1997). This heightened abstraction demands a shift in thinking, challenging students to visualize and understand entities that aren't always concretely presented.

3. Symbolic Language

The language of algebra, dense with symbols and notations, can be another stumbling block. While students might be familiar with basic algebraic notation, the introduction of new symbols, often without clear analogs in their previous

experience, can be confusing (Kieran, 2007). This symbolic language, though concise and efficient for seasoned mathematicians, can act as a barrier for students trying to decode and internalize new concepts.

4. Problem Solving and Application

While students might grasp individual algebraic concepts in isolation, they often face difficulties when required to apply this knowledge in problem-solving scenarios. Translating real-world problems into algebraic expressions, determining the appropriate methods to solve them, and interpreting results in meaningful ways are all common challenges (Lesh & Zawojewski, 2007).

5. Overcoming Previous Misconceptions

Misconceptions or misunderstandings from earlier education can persist into undergraduate studies, hindering progress. For instance, students might hold onto mistaken beliefs about the properties of algebraic operations or the behavior of algebraic functions, leading to consistent errors and frustration (Booth, 1988).

6. Motivation and Anxiety

The challenges of algebra, combined with the pressures of undergraduate study, can lead to decreased motivation and increased math anxiety. Students who face repeated difficulties might develop a belief that they "just aren't good at math," which can become a self-fulfilling prophecy, further exacerbating their challenges (Ashcraft & Moore, 2009).

Some proposals and ideas for designing Case Studies activities in class

Let us discuss some activities for the Teaching and Learning of Algebra at Undergraduate Levels based on Case Studies Pedagogy and Supported by the ATD Theory. Each activity aims to provide students with a multifaceted, immersive experience in algebra, grounding their understanding in real-world contexts, cultural insights, and practical applications. It is essential to underscore that each session is delineated in a structured and succinct format. An extended exposition of each task would culminate in an excessively comprehensive article. Nonetheless, it's posited that an educator with minimal access to assorted resources and tools can craft particular sessions tailored to their scenario based on the furnished descriptions.

1. Cultural Exploration of Algebraic Concepts:

a. *Description:* This activity seeks to immerse students in the historical and cultural evolution of specific algebraic ideas.

b. *Procedure:*

- Begin with a lecture or presentation on the chosen algebraic concept and its cultural origins.
- Distribute case studies detailing the development and application of the concept

in various societies.

- Organize students into discussion groups to dissect the case studies, highlighting similarities and differences in approaches across cultures.
- Conclude with a class-wide discussion, drawing connections between historical/cultural practices and modern algebraic methods.
- Assessment: Students can prepare short essays or presentations summarizing their findings, reflections, and the significance of the cultural context in shaping algebraic thought.

ATD Support: This activity aligns with the ATD's emphasis on viewing mathematical knowledge as a cultural artifact. By exploring the cultural origins and evolution of algebraic concepts, students gain a deeper appreciation of their historical and societal significance.

2. Real-world Mathematical Modeling:

- a. *Description:* Students tackle pressing real-world challenges using algebraic modeling.
- b. *Procedure:*
 - Introduce the real-world problem, providing necessary background and context.
 - Guide students in identifying the key algebraic components or variables related to the problem.
 - Task students with constructing algebraic models that represent the problem's dynamics.
 - Encourage group discussions where students can critique and refine their models.
 - Have students use their models to propose potential solutions or predictions.
 - Assessment: Students can present their models and findings, justifying their approaches and discussing the implications of their results.

ATD Support: Tackling real-world problems enables students to engage with the "praxeological" aspect of the ATD, combining practical tasks with theoretical justifications. This bridges the gap between abstract algebra and its tangible applications.

3. Group Problem-solving with Diverse Techniques:

- a. *Description:* This activity promotes collaborative learning and exposes students to a variety of algebraic techniques.
- b. *Procedure:*
 - Pose a challenging algebraic problem to the class.
 - Assign different historical or cultural algebraic techniques to each group.
 - Allow time for group discussions and problem-solving, using the assigned techniques.

- Organize a class seminar where each group presents their approach and solution.
- Facilitate a class-wide discussion comparing and contrasting the methods, drawing insights from each technique's strengths and weaknesses.
- Assessment: Groups can submit a detailed report on their assigned technique, their problem-solving process, and reflections on the method's applicability and limitations.

ATD Support: This activity emphasizes the diverse "praxeologies" present in algebra, allowing students to experience various practical techniques and the theoretical frameworks behind them.

4. Algebraic Ethnography:

a. *Description:* Students connect with professionals to understand the real-world applications of algebra.

b. *Procedure:*

- Provide guidelines on how to conduct interviews, emphasizing respect, curiosity, and active listening.
- Task students with identifying and reaching out to professionals in algebra-intensive fields.
- After conducting interviews, students should transcribe and analyze their findings, identifying key themes and insights.
- Organize a sharing session where students can present their ethnographies, highlighting the practical uses and challenges of algebra in various professions.
- Assessment: Students submit a comprehensive case study based on their interviews, including a reflection on the significance of algebra in the chosen profession and its broader societal impact.

ATD Support: This activity immerses students in the societal and cultural contexts in which algebraic knowledge is applied, resonating with the ATD's emphasis on the sociocultural dimensions of mathematical activity.

5. Critique and Reconstruction of Algebraic Concepts:

a. *Description:* This activity sharpens students' analytical skills by having them engage with flawed algebraic content.

b. *Procedure:*

- Distribute the case study containing intentional algebraic misconceptions or errors.
- In groups, students should dissect the case study, identifying and noting down the errors.
- Each group should then brainstorm and develop corrected methods or approaches.
- Convene a class discussion where groups share their critiques and corrected methodologies.

- Assessment: Groups can prepare a detailed critique report, highlighting the identified errors, their implications, and the proposed corrected approaches, supported by current algebraic theories and practices.

ATD Support: By engaging with flawed or outdated algebraic knowledge, students are encouraged to delve deeper into the theoretical justifications of current algebraic methods, aligning with the ATD's focus on the interconnectedness of practical tasks and theoretical understanding. It explores the potential of active pedagogy to enhance students' understanding and appreciation of algebra.

RESEARCH METHOD

To collect robust student feedback a semi-structured interview method is employed. This is a qualitative research method characterized by its flexibility and depth, allowing researchers to delve into specific topics while also permitting unexpected avenues of discovery. Unlike structured interviews, which rely on a fixed set of questions, semi-structured interviews employ an interview guide with open-ended questions, facilitating a more conversational and responsive interaction between the interviewer and the participant (Brinkmann & Kvale, 2015). This flexibility ensures that while certain key topics are addressed, the participant also has the freedom to express their perspectives, experiences, and insights in their own words, leading to richer data collection.

The semi-structured interview is particularly beneficial when exploring complex or nuanced topics, where participants' personal experiences, feelings, and perspectives are paramount. It offers a balance between the standardization of questions, ensuring consistency across interviews, and the freedom to probe deeper or diverge based on participants' responses (DiCicco-Bloom & Crabtree, 2006). Furthermore, this method provides ample opportunity to clarify and understand the context behind participants' responses, capturing the depth, intensity, and intricacies of their perspectives (Galletta, 2013).

However, the flexibility of semi-structured interviews also poses challenges. It requires skilled interviewers who can navigate the balance between following the guide and allowing spontaneous exploration. They should be adept at active listening, ensuring participants feel heard and understood, and be proficient in probing techniques to elicit detailed responses without leading the participant (Turner, 2010).

RESULTS AND DISCUSSION

We now delve into the advantages and limitations of each activity, shedding light on the multifaceted nature of algebraic education and its impact on student understanding and appreciation. The feedback is presented in a concise manner after blending with the different feedbacks compiled.

1. Cultural Exploration of Algebraic Concepts Feedback:

- a. Pros: This activity offered a fresh perspective on algebra, connecting history with mathematics in a manner that was both enlightening and engaging. It was intriguing to realize that the algebraic concepts we study have diverse cultural origins and have evolved over time. The discussion groups were a productive space for sharing and analyzing different cultural methods.
 - b. Cons: Some case studies were more complex than others, which led to varying discussion depths across groups. A little more guidance on dissecting the case studies might have been helpful.
2. Real-world Mathematical Modeling Feedback:
- a. Pros: The hands-on nature of this activity helped in grasping the real-world applicability of algebraic models. Working on contemporary issues made the task more relevant and pressing, and the group discussions allowed for a range of solutions and perspectives.
 - b. Cons: Depending on the problem presented, some groups found it challenging to identify the key algebraic components, which affected the rest of the task. Providing a few more examples or initial guidance could improve this aspect.
3. Group Problem-solving with Diverse Techniques Feedback:
- a. Pros: The activity was insightful, showcasing the breadth and variety of algebraic techniques across cultures and histories. Collaborative problem-solving fostered teamwork and allowed for collective brainstorming.
 - b. Cons: Some assigned techniques were harder to grasp and apply than others. Perhaps a brief introductory session or resources for each technique could help level the playing field.
4. Algebraic Ethnography Feedback:
- a. Pros: This was an eye-opening activity that offered a chance to connect with professionals and understand algebra's practical implications. Conducting interviews gave a real-world touch and made the concepts more tangible.
 - b. Cons: Not everyone was able to secure interviews with professionals in algebra-intensive fields, making it a bit uneven in terms of experience. A backup option or a list of potential interviewees might be beneficial.
5. Critique and Reconstruction of Algebraic Concepts Feedback:
- a. Pros: This activity was an excellent exercise in critical thinking, encouraging us to challenge and reassess our existing algebraic knowledge. Identifying and correcting errors was both challenging and rewarding.
 - b. Cons: Some of the intentional misconceptions were either too subtle or too complex, making them hard to spot or correct without additional guidance. A hint system or checkpoints might enhance the learning experience.

The five algebraic activities offer diverse pedagogical approaches to deepen students' understanding and appreciation of algebra. From exploring algebra's

cultural and historical facets to tackling real-world problems with mathematical models, students are granted a comprehensive learning experience. Engaging with professionals through interviews reveals the practical utility of algebra, while critical analysis of flawed content strengthens analytical abilities. Although these activities are generally enlightening and engaging, certain challenges, such as uneven complexity across groups or the need for additional guidance, emerge. Overall, these tasks effectively intertwine algebraic learning with practical, historical, and analytical components, aligning well with the ATD's emphasis on the interconnectedness of practical tasks and theoretical understanding.

In the ever-evolving landscape of STEM education, the role of methodology case studies, combined with theoretical frameworks like the Anthropological Theory of Didactics (ATD), is pivotal in enhancing the teaching and learning of critical subjects like algebra. Through a deep dive into the intricacies of methodology application, educators and students alike gain invaluable insights into the challenges and nuances of practical implementation in STEM subjects.

The ATD, with its anthropological lens, offers a revolutionary perspective on the teaching and learning of mathematics. By understanding mathematical knowledge as a cultural endeavor, it shifts the spotlight from mere content mastery to a holistic appreciation of the historical, cultural, and societal contexts that shape mathematical concepts. This perspective, when combined with active pedagogy, presents a transformative approach to algebra education, emphasizing real-world applications, collaborative problem-solving, and a deep understanding of cultural and historical contexts.

Furthermore, by addressing the challenges faced by undergraduate students in algebra, educators can tailor their teaching methods to better cater to student needs. The integration of real-world case studies, cultural explorations, and collaborative activities not only makes algebra more accessible but also fosters a sense of curiosity, engagement, and deep understanding among students.

Although the proposed activities are generally enlightening and engaging, certain challenges, such as uneven complexity across groups or the need for additional guidance, emerge based on the students' feedback. Overall, these tasks effectively intertwine algebraic learning with practical, historical, and analytical components, aligning well with the ATD's emphasis on the interconnectedness of practical tasks and theoretical understanding.

CONCLUSION

In the convergence of active pedagogy case studies and the ATD, we find a promising avenue for the future of mathematics education. This integrated approach not only ensures that students are active participants in their learning journey but also equips them with a culturally grounded, comprehensive understanding of mathematical concepts. As we move forward, it is essential to continue exploring and integrating such innovative methodologies and theories, ensuring that STEM education remains relevant, engaging, and effective in preparing students for the challenges and opportunities of the 21st century.

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