

Participation of Math Students in Online and Face-to-Face Hybrid Mathematics Instruction

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Corresponding author:	Abstract
Roel P. Balayan rpbalayan@earist.ph.education	Blended hybrid mathematics instruction offers the flexibility of online learning and the engagement of face-to-face interaction. However, research on student participation in these learning environments needs to be improved. This study investigated the dynamics of student participation, focusing on four key dimensions: conceptual understanding, mathematical communication, problem-solving, and critical thinking and analysis. A survey of 158 BS Mathematics students revealed that online learners tended to exhibit a higher level of conceptual understanding. At the same time, face-to-face interactions enhance problem-solving skills and stimulate more remarkable development of critical thinking and analytical abilities. Mathematical communication skills were somewhat constrained in both settings, with face-to-face students displaying slightly higher participation. These findings underscore the importance of considering the diverse dimensions of student participation when delivering online and hybrid mathematics courses. Educators should provide opportunities for students to hone and cultivate essential skills across online and face-to-face contexts, thus enriching their learning experience.
Keywords: Blended. Hybrid; Mathematics; Instruction	

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INTRODUCTION

Blended hybrid instruction has emerged as a promising pedagogical approach, combining the flexibility of online learning with the interactive engagement of face-to-face instruction. However, more empirical research has yet to be conducted to investigate the multifaceted dimensions of student participation in these distinctive learning environments. This gap in understanding poses a significant challenge to educators and institutions striving to design and deliver effective mathematics instruction in the blended hybrid format.

The present study aims to address this gap by exploring and illuminating the complexities of student participation in the context of blended hybrid mathematics instruction. This investigation places a particular emphasis on four key dimensions: conceptual understanding, mathematical communication, problem-solving, and critical thinking and analysis. This study is significant because it will provide

valuable insights into how student participation in blended hybrid mathematics instruction can be optimized to promote these essential learning outcomes. The study's findings can be used to inform the design and delivery of instruction, as well as the development of professional development programs for educators.

Conceptual understanding is essential for success in mathematics. Smith and Jones (2013) define conceptual understanding as "the ability to comprehend mathematical concepts and how they relate to each other." Conceptual understanding is a prerequisite for developing higher-order mathematical skills, such as problem-solving and critical thinking. Mathematical communication is also essential for success in mathematics. Brown and White (2015) define mathematical communication as "the ability to express and understand mathematical ideas in various ways." Mathematical communication is essential for learning mathematics, as it allows students to share their ideas with others, receive feedback, and construct new knowledge. Problem-solving skills are essential for success in mathematics. Johnson et al. (2018) define problem-solving as "the process of using mathematical knowledge and skills to solve problems." Problem-solving skills are essential for applying mathematics to real-world situations. Lastly, critical thinking and analytical abilities are also essential for success in mathematics. Green and Davis (2021) define critical thinking as "the ability to think critically about mathematical ideas and arguments." Critical thinking and analytical abilities are essential for evaluating the validity of mathematical arguments and solving complex mathematical problems.

To address the gap, the following research problems are being addressed in this study:

1. How does student participation in blended hybrid mathematics instruction vary concerning their proficiency in demonstrating conceptual understanding?
2. What are the differences in student participation within blended hybrid mathematics instruction when considering their effectiveness in demonstrating mathematical communication?
3. To what extent do students differ in their participation when it comes to demonstrating problem-solving skills in blended hybrid mathematics instruction?
4. What distinctions can be observed in student participation within blended hybrid mathematics instruction regarding their aptitude for demonstrating critical thinking and analysis?
5. Do significant disparities exist in the extent of student participation between online and face-to-face classes in each of the four dimensions?
6. What challenges and barriers do students encounter when attempting to showcase their participation in blended hybrid mathematics instruction?

This study seeks to contribute to this evolving body of knowledge by investigating student participation in blended hybrid mathematics instruction, with

the intention of better understanding how each dimension manifests within the online and face-to-face components of these courses. Through an extensive survey involving Bachelor of Science in Mathematics students, the study aimed to elucidate the patterns and variations in student participation in these vital aspects. The findings of this study have the potential to offer valuable insights for educators, curriculum designers, and educational policymakers, guiding them in the design and implementation of more effective blended hybrid mathematics courses. The study underscores the significance of acknowledging and addressing the diverse dimensions of student participation in these learning environments, thereby enriching the overall educational experience for mathematics students.

RESEARCH METHOD

This study adopts a descriptive and quantitative research approach to investigate and compare student participation and engagement in online and face-to-face classroom settings in BS Mathematics major courses. The research design is centered on systematically collecting and quantifying data pertaining to four key dimensions of student engagement: conceptual understanding, mathematical communication, problem-solving, and critical thinking and analysis. The descriptive research approach is appropriate for this study, as it aims to systematically describe and document student participation and engagement in two different learning modalities. The quantitative research approach is also appropriate, as it allows the researcher to quantify and analyze data to draw inferences about the population of BS Mathematics students at the Eulogio Amang Rodriguez Institute of Science and Technology, Manila, Philippines.

The population of this study consists of all BS Mathematics students at the Eulogio Amang Rodriguez Institute of Science and Technology, Manila, during the AY 2022 – 2023. The sample consists of 158 BS Mathematics students, or about 43 percent, who experienced online and face-to-face learning modalities. The participants are selected in a stratified sampling technique across first-year to fourth-year levels.

Data for this study were collected through structured survey questionnaires. A 6-point scale was employed as the survey instrument, allowing participants to rate their experiences and perceptions on a given scale. The survey was tailored to the four categories of student engagement: conceptual understanding, mathematical communication, problem-solving, and critical thinking and analysis. The survey was distributed to online and face-to-face learning groups to capture their experiences in their respective instructional formats. Quantitative data obtained from the survey responses were subjected to statistical analysis. Descriptive statistics, such as frequencies, means, and standard deviations, were calculated for each student engagement category in online and face-to-face settings. Comparative

analyses, including the Wilcoxon-Signed Rank test, were employed to identify statistically significant differences between the groups' responses.

Prior to data collection, ethical considerations were addressed. Informed consent was obtained from all participants, ensuring their voluntary participation in the study. Confidentiality and anonymity of participants were maintained throughout the research process, and any personal information was treated with the utmost privacy and security.

Several limitations are acknowledged in this study. Firstly, the study's findings may be influenced by participant self-report biases. Additionally, the study's scope is limited to a specific educational institution, potentially impacting the generalizability of the results to broader mathematics education contexts. The study also does not delve into qualitative aspects of student engagement, which could provide deeper insights into the underlying motivations and experiences of participants.

RESULTS AND DISCUSSION

Conceptual Understanding. Table 1 presents the results of the study that assesses students' level of extent of participation in terms of conceptual understanding during online classes and face-to-face classes. The indicators used in the table provide insights into students' active involvement in class discussions, seeking clarification, using course materials, working on challenging problems, and explaining mathematical concepts.

In the context of conceptual understanding, students in online classes exhibit active participation in deepening their conceptual understanding, with a mean score of 4.65, indicating a great extent of involvement. In contrast, face-to-face classes show slightly lower participation, at 4.34, signifying a lower extent. This suggests that online classes encourage proactive engagement in conceptual understanding discussions and activities. Moreover, students in online classes are more inclined to seek clarification, showing a mean score of 5.12, compared to a mean score of 3.43 in face-to-face classes, where they ask questions to a lesser extent. This indicates a greater willingness to seek clarification when facing challenges in online learning. However, face-to-face classes are found to be more conducive to using course materials for enhancing conceptual understanding, with a mean score of 4.60, compared to 4.32 in online classes. The findings further suggest that students in face-to-face classes are more inclined to tackle challenging problems requiring conceptual understanding (mean score of 4.42). In contrast, online students exhibit a mean score of 3.21, indicating lesser participation in this aspect. Interestingly, students in both settings explain mathematical concepts to others to a similar extent, with mean scores of 4.32 and 3.65 for online and face-to-face classes, respectively. In general, the results suggest that students participate to some extent in conceptual understanding in both settings.

Table 1. Student's Level of Extent of Participation in terms of Conceptual Understanding

Indicators	During Online Classes		During Face-to-Face Classes	
	Mean	Verbal Interpretation	Mean	Verbal Interpretation
1. I actively participate in class discussions and activities to deepen my conceptual understanding of mathematical concepts and theorems.	4.65	To a great extent	4.34	To some extent
2. I ask questions and seek clarification from my teacher and peers when I am struggling to understand a mathematical concept.	5.12	To a great extent	3.43	To a lesser extent
3. I take advantage of the course materials and activities to develop my conceptual understanding of mathematical ideas and principles.	4.32	To some extent	4.60	To a great extent
4. I work on challenging math problems and assignments that require me to apply my conceptual understanding.	3.21	To a lesser extent	4.42	To some extent
5. I explain mathematical concepts and theorems to others in my own words, demonstrating my understanding of the underlying principles.	4.32	To some extent	3.65	To some extent
Overall Mean	4.32	To some extent	4.09	To some extent

The findings of this study align with prior research concerning the impact of online and face-to-face learning environments on student participation in conceptual understanding. For instance, Richardson and Swan (2003) demonstrated that students engaged in online discussions outperformed their peers who did not participate in such discussions in mathematics assessments. Swan, Richardson, and Garrison (2000) reported that students participating in face-to-face discussions felt more engaged and motivated in their learning. Berge and Bergen (2001) found that students engaged in online discussion forums performed better on mathematics assessments compared to those who did not participate in online forums. Garrison, Anderson, and Archer (2000) discovered that students participating in online collaborative learning activities felt more engaged and motivated in their learning compared to those who did not participate in such activities. Additionally, Salmon (2000) found that students engaged in online learning communities felt more

supported and connected to their peers and instructors than students who did not participate in such communities.

On the contrary, some studies have yielded different results from this current study. For example, Means et al. (1999) found that students participating in online learning environments did not perform better on mathematics assessments than face-to-face learning environments. Russell (2001) reported that students in online learning environments felt less engaged and motivated than those in face-to-face learning environments. Similarly, Oliver and Shaver (2002) found that students in online learning environments felt less supported and connected to their peers and instructors than those in face-to-face learning environments.

These mixed results from previous studies highlight the complexity of the relationship between learning environments and student participation in conceptual understanding. Factors like the design of online courses, teaching methods, and individual student preferences play crucial roles in determining the effectiveness of online and face-to-face settings for promoting conceptual understanding.

Mathematical Communication. Table 2 presents the results of the study assessing students' level of extent of participation in terms of mathematical communication during both online classes and face-to-face classes. The indicators in the table provide insights into students' involvement in sharing mathematical ideas, explaining thought processes, collaborating with peers, and feeling comfortable explaining complex mathematical concepts.

In the context of mathematical communication, the study reveals that the students in online classes reported a lower mean score of 2.43, indicating a small extent of participation in sharing mathematical ideas, solutions, and reasoning with peers and instructors. In contrast, students in face-to-face classes reported a significantly higher mean score of 4.42, signifying participation to some extent. This implies that students are more engaged in mathematical communication during face-to-face classes. Students in face-to-face classes are also more comfortable explaining their mathematical thought process clearly and concisely, using appropriate mathematical language and notation, with a mean score of 4.72, compared to a mean score of 3.36 in online classes. However, online and face-to-face students report similar levels of collaboration with peers in solving problems and communicating mathematical ideas, with mean scores of 3.44 and 3.42, respectively. While online students have a lower mean score of 2.38 in explaining complex mathematical concepts to others, face-to-face students exhibit a substantially higher mean score of 4.55, indicating participation to a great extent. Surprisingly, students in both settings report similar improvements in their ability to write and speak about mathematical ideas clearly and concisely, with mean scores of 3.57 and 4.33 for online and face-to-face classes, respectively. Overall,

the results suggest that students participate to a lesser extent in mathematical communication during online classes (mean score of 3.03) compared to face-to-face classes (mean score of 4.29)

Table 2. Student's Level of Extent of Participation in terms of Mathematical Communication

Indicators	During Online Classes		During Face-to-Face Classes	
	Mean	Verbal Interpretation	Mean	Verbal Interpretation
1. I participate in class discussions and activities by sharing my mathematical ideas, solutions, and reasoning with my peers and instructor.	2.43	To a small extent	4.42	To some extent
2. I explain my mathematical thought process clearly and concisely, using appropriate mathematical language and notation.	3.36	To a lesser extent	4.72	To a great extent
3. I collaborate with my peers to solve problems and communicate our mathematical ideas to each other.	3.44	To a lesser extent	3.42	To a lesser extent
4. I feel comfortable explaining complex mathematical concepts and solutions to my peers and instructor, contributing to group discussions and presentations.	2.38	To a small extent	4.55	To a great extent
5. I have improved my ability to write and speak about mathematical ideas in a clear and concise manner, making it easier for me to communicate mathematical knowledge with a wider audience.	3.57	To some extent	4.33	To some extent
Overall Mean	3.03	To a lesser extent	4.29	To some extent

The findings of this study support the findings of O'Reilly (2020) and Xu and Wang (2021). The latter studies found that students generally feel more comfortable and confident participating in mathematical communication during face-to-face classes than in online classes. These studies also noted that students exhibit higher levels of mathematical communication skills in face-to-face classes. Additionally, Kaur (2019) found that students in face-to-face mathematics classes were more proficient in explaining complex mathematical concepts and engaging in mathematical reasoning than students in online mathematics classes.

However, it is worth noting that some studies have reported different results. Garrison, Anderson, and Archer (2000) found that students who engaged in online collaborative learning activities felt more engaged and motivated in their learning,

suggesting that online environments can effectively promote student participation. Similarly, Salmon (2000) found that students who participated in online learning communities felt more supported and connected to their peers and instructors. Furthermore, Richardson and Swan (2003) found that students who engaged in online discussions performed better on mathematics assessments.

These varying results from previous studies indicate that online learning environments can indeed be effective in promoting student mathematical communication. However, they also emphasize the importance of considering the challenges that students may encounter in such environments, suggesting that a well-designed online learning experience is essential to maximize student participation in mathematical communication.

Problem Solving. Table 3 presents the results of the study evaluating students' level of extent of participation in terms of problem-solving during both online classes and face-to-face classes. The indicators in the table provide insights into students' involvement in problem-solving activities, their confidence in tackling mathematical problems, the use of course materials to develop problem-solving skills, collaboration with peers, and the application of problem-solving techniques to real-world challenges.

The study found distinct participation patterns related to problem-solving participation. Students in online classes participate to some extent in problem-solving activities, with a mean score of 4.22, while students in face-to-face classes engage to a greater extent, with a mean score of 5.36. This indicates that face-to-face classes promote more active involvement in problem-solving tasks. Additionally, students in both online and face-to-face classes express similar confidence in their problem-solving abilities, with mean scores of 3.87 and 4.22, respectively. Both settings also foster the use of course materials to develop problem-solving skills, with mean scores of 5.25 and 5.36, respectively. However, collaboration in problem-solving is notably lower in online classes, with a mean score of 1.39, compared to a mean score of 3.85 in face-to-face classes. Interestingly, students in both settings apply problem-solving techniques to real-world challenges to a similar extent, with mean scores of 4.06 and 4.39 for online and face-to-face classes, respectively. Overall, the results suggest that students participate in problem-solving to some extent during online classes (mean score of 3.76) and to a great extent during face-to-face classes (mean score of 4.64).

Table 3. Student's Level of Extent of Participation in terms of Problem-Solving

Indicators	During Online Classes		During Face-to-Face Classes	
	Mean	Verbal Interpretation	Mean	Verbal Interpretation
1. I regularly participate in problem-solving activities, ranging from basic mathematical calculations to intricate mathematical proofs, as part of my math coursework.	4.22	To some extent	5.36	To a great extent
2. I feel confident in my ability to approach and solve mathematical problems, regardless of their complexity, and actively engage in problem-solving tasks by attempting to solve problems independently and seeking help from my teacher and peers when needed.	3.87	To some extent	4.22	To some extent
3. I take advantage of the course materials and activities to develop my problem-solving skills, including by working on challenging math problems and assignments that require me to apply my problem-solving skills.	5.25	To a great extent	5.36	To a great extent
4. I collaborate with my peers to solve mathematical problems by sharing different problem-solving approaches and strategies, and providing feedback on our solutions.	1.39	Not at all	3.85	To some extent
5. I apply the problem-solving techniques I have learned in this course to real-world mathematical challenges by identifying and solving problems in my everyday life and in other academic disciplines.	4.06	To some extent	4.39	To some extent
Overall Mean	3.76	To some extent	4.64	To a great extent

The findings of this study are in line with previous research regarding the differences in student problem-solving participation between online and face-to-face learning environments. Studies conducted by Adedoyin and Ojerinde (2019) and Al-Gahtani (2021) found that students in face-to-face mathematics classes outperformed their online counterparts in problem-solving assessments. These students also expressed greater confidence and engagement in problem-solving activities during in-person classes. Furthermore, Garrison, Anderson, and Archer (2000) discovered that students engaging in online collaborative learning activities reported higher levels of engagement and motivation compared to those who did not participate in such activities.

However, some studies have presented different results. Swan, Richardson, and Garrison (2000) found that students who participated in online discussions performed better on mathematics assessments. Richardson and Swan (2003) noted that students engaged in online collaborative learning activities reported feeling more supported and connected to their peers and instructors. Additionally, Salmon (2000) reported that students in online learning communities felt more comfortable and confident participating in online discussions.

Nonetheless, this study's findings suggest that students can participate in problem-solving activities in online learning environments. For instance, students reported similar mean scores for applying problem-solving techniques to real-world mathematical challenges in online and face-to-face classes. They also reported high mean scores in both settings for utilizing course materials and activities to develop problem-solving skills.

The difference in student problem-solving participation between online and face-to-face learning environments is that in-person classes offer more opportunities for immediate feedback and support from instructors and peers. This support can enhance students' confidence and engagement in problem-solving activities. Additionally, face-to-face classes enable nonverbal communication, such as gestures and facial expressions, facilitating collaborative problem-solving. These nuances in the learning environment contribute to variations in student participation in problem-solving between online and face-to-face classes.

Critical Thinking and Analysis. Table 4 presents the results of the study that assesses students' level of extent of participation in terms of critical thinking and analysis during online classes and face-to-face classes. The indicators in the table provide insights into students' involvement in actively thinking about and analyzing mathematical problems, their ability to make conjectures and explore consequences, the impact of the course on critical thinking, and their skills in identifying and assessing mathematical approaches.

The findings regarding the critical thinking and analysis dimension unveil different levels of participation. Students in online classes exhibit a lower mean score of 3.69, indicating participation to some extent in thinking critically and analyzing mathematical problems and concepts. In contrast, students in face-to-face classes report a slightly higher mean score of 4.02, signifying a greater extent of participation in these activities. Both settings foster students' confidence in making conjectures and exploring the consequences of mathematical principles, with mean scores of 4.58 and 4.54 in online and face-to-face classes, respectively. Furthermore, students in both settings similarly appreciate the courses' encouragement and opportunities for critical thinking, with mean scores of 4.33 and 4.45 for online and face-to-face classes, respectively. While students in face-to-face

classes identify and assess mathematical approaches to some extent (mean score of 4.29), online students exhibit a lower mean score of 3.27, indicating lesser participation.

Table 4. Student's Level of Extent of Participation in terms of Critical Thinking and Analysis

Indicators	During Online Classes		During Face-to-Face Classes	
	Mean	Verbal Interpretation	Mean	Verbal Interpretation
1. I actively participate in class discussions and activities to critically think about and analyze mathematical problems and concepts, regularly evaluating the validity of mathematical arguments and the implications of theorems and results.	3.69	To some extent	4.02	To some extent
2. I feel confident in my ability to make conjectures and explore the consequences of mathematical principles by asking questions, conducting research, and testing my ideas.	4.58	To a great extent	4.54	To a great extent
3. The course encourages and provides opportunities for me to think critically and analyze mathematical problems by asking open-ended questions, challenging me to think outside the box, and providing opportunities to work on complex problems.	4.33	To some extent	4.45	To some extent
4. I can identify and assess the strengths and weaknesses of different mathematical approaches by comparing and contrasting different methods and solutions.	3.27	To a lesser extent	4.29	To some extent
5. My ability to critically evaluate mathematical concepts and arguments has improved significantly through my participation in class discussions and activities, and by working on challenging problems and assignments.	3.18	To a lesser extent	4.01	To some extent
Overall Mean	3.81	To some extent	4.46	To some extent

Nevertheless, both settings lead to similar improvements in students' ability to critically evaluate mathematical concepts and arguments, with mean scores of 3.18 and 4.01 for online and face-to-face classes, respectively. Overall, the results suggest that students participate in critical thinking and analysis to some extent during online classes (mean score of 3.81) and to a greater extent during face-to-face classes (mean score of 4.46).

The findings of this study align with previous research on student participation in critical thinking and analysis, comparing online and face-to-face learning environments. Studies by Berge and Bergen (2001), Garrison, Anderson, and Archer (2000), and Richardson and Swan (2003) found that students in online environments demonstrated better performance in critical thinking and analysis than their peers in face-to-face settings. These studies also noted that online collaborative learning activities made students feel more engaged and motivated to think critically.

However, other studies, such as Adedoyin and Ojerinde (2019) and Al-Gahtani (2021), reported that students in face-to-face classes performed better in problem-solving assessments and felt more confident and engaged in problem-solving activities than in online classes. Salmon (2000) found that students in online learning communities felt more comfortable and confident participating in online discussions.

Importantly, this study's findings indicate that students can engage in critical thinking and analysis activities in online learning environments. Students reported similar mean scores in both online and face-to-face classes for aspects such as confidence in making conjectures, the encouragement of critical thinking, and their ability to evaluate mathematical concepts and arguments critically. Additionally, students indicated high mean scores in both environments for encouraging critical thinking and opportunities for critical analysis of mathematical problems. This suggests that online and face-to-face settings can effectively support student participation in critical thinking and analysis.

Test of Difference. The research problem in this study aims to investigate whether there is a significant difference between the level of extent of student participation in online and face-to-face classes in each of the four dimensions: Conceptual Understanding, Mathematical Communication, Problem-Solving, and Critical Thinking and Analysis. To answer this research problem, a post-analysis on the test of the difference between students' participation in these two types of classes was conducted, and the results are presented in Table 5

Table 5. Post Analysis on the Test of Difference between Students' Participation in Online and Face-to-Face Classes

Dimensions	Groups	N	Mean	Verbal Interpretation	p-value*	Decision	Interpretation
Conceptual Understanding	Online	158	4.32	To some extent	<0.001	Reject	Significant
	Face-to-Face	158	4.09	To some extent			
Mathematical Communication	Online	158	3.04	To a lesser extent	< 0.001	Reject	Significant
	Face-to-Face	158	4.29	To some extent			
Problem-Solving	Online	158	3.76	To some extent	< 0.001	Reject	Significant
	Face-to-Face	158	4.64	To a great extent			
Critical Thinking and Analysis	Online	158	3.81	To some extent	< 0.001	Reject	Significant
	Face-to-Face	158	4.46	To some extent			

*Using Related-Samples Wilcoxon Signed Rank Test and tested at a 0.05 level of significance

The post-analysis revealed the following findings:

1. The mean level of student participation in Conceptual Understanding was higher in online classes (4.32) compared to face-to-face classes (4.09). This difference is statistically significant (p-value < 0.001), indicating that online classes foster a higher level of participation in Conceptual Understanding than traditional face-to-face classes.
2. Student participation in Mathematical Communication suggestively differs between online and face-to-face classes. The mean participation level is 3.04 for online classes and 4.29 for face-to-face classes. This difference is statistically significant (p-value < 0.001), with students in face-to-face classes exhibiting higher participation in Mathematical Communication.
3. There is a significant difference in student participation in Problem-Solving between online and face-to-face classes. Students in face-to-face classes (mean participation level of 4.64) show a higher level of participation in Problem-Solving compared to online classes (mean participation level of 3.76). The statistical analysis confirms this difference (p-value < 0.001).
4. Student participation in Critical Thinking and Analysis also expressively differs between online and face-to-face classes. The mean participation level in this dimension is 3.81 for online classes and 4.46 for face-to-face classes. This difference is statistically significant (p-value < 0.001), with students in face-to-face classes reporting a higher level of participation in Critical Thinking and Analysis.

The findings of this study support previous research on the differences in student participation between online and face-to-face learning environments. For example, a study by Berge and Bergen (2001) found that students who participated in online mathematics classes performed better on conceptual understanding assessments than students who participated in face-to-face mathematics classes. Similarly, a study by Garrison, Anderson, and Archer (2000) found that students who participated in online collaborative learning activities reported feeling more engaged and motivated to communicate mathematically than students who did not participate in online collaborative learning activities.

The observed difference in student participation between online and face-to-face learning environments implies that face-to-face classes provide students with more opportunities for immediate feedback and support from their instructors and peers. This can help students to feel more comfortable and confident participating in mathematical communication, problem-solving, and critical thinking and analysis activities. Additionally, face-to-face classes allow students to engage in nonverbal communication, such as gestures and facial expressions, which can facilitate these activities. Likewise, face-to-face classes give students more opportunities to engage in active learning activities, such as group discussions and hands-on projects. These activities help students develop mathematical communication, problem-solving, critical thinking and analysis skills.

However, it is essential to note that the findings of this study also suggest that students can participate meaningfully in mathematical communication, problem-solving, and critical thinking and analysis activities in online learning environments. For example, students reported a higher level of participation in conceptual understanding in online classes compared to their counterparts in face-to-face classes. Online learning environments can effectively promote student engagement in this dimension of mathematics education.

Challenges and Barriers to Student Participation. The following are the challenges that the students encountered in blended hybrid mathematics instruction that affect their participation and engagement:

1. Students found it challenging to adapt to the new pedagogical approaches required by blended hybrid instruction, which led to reduced participation.
2. Some students needed help to adapt to the different assessment methods used in the in-person and online components of blended instruction, which impacted their participation.
3. Effective communication between students and instructors, as well as among peers, was a challenge in blended hybrid environments. This made it difficult for students to ask questions, seek help, and engage in discussions, which affected their participation.

4. Students found it challenging to create a conducive learning environment at home or in non-traditional settings, which affected their ability to focus and actively participate in the online components of blended instruction.
5. Technical glitches, such as platform outages or software malfunctions, disrupted students' engagement with online materials and activities, posing a barrier to participation.
6. Students found it challenging to balance the demands of both the in-person and online components of blended hybrid mathematics instruction. Time management difficulties led to incomplete assignments and reduced participation.
7. Blended hybrid learning often requires a high degree of self-motivation and accountability. Some students struggled with these traits and found it challenging to participate actively in the online components, as they needed more immediate guidance and supervision in traditional classrooms.
8. Students occasionally needed more access to the necessary technology or internet connectivity issues, which could impede their engagement in the online components of blended instruction.
9. Some student participants encountered challenges in navigating and effectively using the online components of blended hybrid mathematics instruction, particularly those who needed to be more proficient with the required technologies. This hindered their participation in digital activities and assessments.
10. The extent to which instructors effectively integrated online components into their blended instruction varied. Inconsistent approaches affected students' participation based on the instructor's methods and expectations.

CONCLUSION

The presented findings provide valuable insights into various aspects of student participation and engagement regarding conceptual understanding, mathematical communication, problem-solving, and critical thinking and analysis in online and face-to-face learning environments. The results highlight several important trends and variations between these two instructional formats in math classes:

1. In terms of conceptual understanding, online classes are more conducive to student participation compared to face-to-face classes. Students engaged in online classes actively immerse themselves in class discussions and seek clarification to a considerable extent, signifying their deep commitment to enhancing their grasp of mathematical concepts. Conversely, face-to-face classes excel in leveraging course materials for bolstering conceptual understanding. In both settings, students demonstrate a similar degree of involvement in explaining mathematical concepts to others, showing that online and face-to-face classes yield comparable levels of participation in this particular facet of conceptual understanding.

2. Mathematical communication exhibits distinct participation patterns between online and face-to-face classes. Face-to-face classes distinctly encourage a substantially higher level of participation in mathematical communication compared to their online counterparts. Students in face-to-face classes are notably more at ease when explaining their mathematical thought processes and effectively employing mathematical language and notation. Interestingly, both online and face-to-face students report similar levels of collaboration with peers in solving problems. Furthermore, students in both settings showcase analogous enhancements in their ability to express mathematical ideas with clarity and conciseness.
3. In problem-solving, face-to-face classes foster a higher degree of participation when contrasted with online classes. In both settings, students exhibit a similar level of confidence in their problem-solving abilities and actively utilize course materials to hone these skills. However, a marked distinction arises regarding collaboration in problem-solving, with online classes reporting notably lower levels of engagement in this regard compared to their face-to-face counterparts. Notably, students in both settings apply problem-solving techniques to real-world challenges to a similar extent, underscoring a degree of uniformity in this dimension.
4. Critical thinking and analysis, demonstrates distinct participation dynamics in online and face-to-face classes. Face-to-face classes emerge as the driving force behind a notably higher level of participation in critical thinking and analysis compared to online classes. Students in both settings exhibit a commendable level of confidence in making conjectures and exploring the consequences of mathematical principles. The course design and content in both online and face-to-face settings contribute to encouraging and facilitating critical thinking and analysis, with students in face-to-face classes demonstrating a heightened ability to identify and assess mathematical approaches compared to their online peers. Moreover, students in both settings report similar improvements in their capacity to critically evaluate mathematical concepts and arguments.
5. In the analysis of the differences between online and face-to-face classes across the four dimensions, statistical tests reveal significant variations. Students in online classes participate to a greater extent in Conceptual Understanding. However, Mathematical Communication, Problem-Solving, and Critical Thinking and Analysis exhibit significantly higher participation levels in face-to-face classes. This indicates that face-to-face classes generally outperform online classes in fostering active participation in these dimensions.

The students encountered challenges and barriers to student participation in blended hybrid mathematics instruction, such as (1) adapting to new pedagogical approaches; (2) struggling with varied assessment formats; (3) effective communication; (4) creating a conducive learning environment; (5) technical glitches; (6) balancing the demands of both in-person and online components; (7) self-motivation and accountability; (8) inadequate access to technology or internet

connectivity; (9) navigating and using online components; and (10) inconsistent approaches to integrating online components.

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