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Misconceptions and Errors Among Grade 12 Students When Learning Differentiation Rules: A Case Study

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Abstract

It is an established fact that some grade 12 students learning differentiation in calculus are found struggling with the rule of differentiation and these rules include power rule, quotient rule, chain rule and product rule. Because of these, some students came up with some misconceptions which eventually resulted to the students having multiple errors when learning rules of differentiation. The reason associated with these common errors are not far from their failure to model some prerequisite knowledge in the laws of logarithm into their learning of the laws of differentiation. And this has resulted to the poor performance of some students in mathematics (calculus) since calculus carries about 40% in the overall grade in mathematics as a subject at grade 12 level. On this note, this paper presents an analysis of students' errors and misconceptions in learning differentiation rule. The study was conducted among grade 12 students preparing for NSC examination in a high school in Limpopo province in South Africa. And the data was initially collected using 35 test scripts of grade 12 on the topic of differentiation and differentiation rule. A qualitative approach was considered, and the data collected was analysed, focusing on product rules, quotient rules and chain rules and the errors committed. The result of the study indicated that some grade 12 students make some errors and misconceptions in differentiation rules. And these were because of poor conceptual understanding, poor mathematics language understanding, and some other error. It was confirmed that the results of this study highlight the common mistakes and errors students make when learning differentiation rules, and these errors are; conceptual, systemic, language, and generalization errors.

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INTRODUCTION

It empirically confirmed that there are common misconceptions and errors that grade 12 students often encountered when learning differential rules. These misconceptions could hinder their understandings and applications of differentiation concepts in real-life situations. It is on this note that some researchers reported some common misconceptions and errors committed among high school students (Jameson et al., 2023; 2024; Strang, 2020; Makonye & Luneta, 2014). And some of these errors are: Procedural and systemics errors in differentiation rules- which is a form of error frequently committed by grade 12 students when struggling with the application of differentiation rules correctly, and this error arises when the students failed to use the quotient rule appropriately when finding derivatives of

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fractions. In addition to this, when applying a chain Rule, there is tendency of misapplying the chain rule, especially when working on a composite function, and this could result to a misconception among them.

Similarly, some researchers confirmed that when learning a power rule, some errors are committed among the grade 12 students due to mistakes making among them, such as adding one to the power instead of subtracting, incorrectly thereby bringing down the power, or failing to find the required derivative, and this eventually led to misconception among them (Chikwanha et al., 2022; Cline et al., 2020). And these misconceptions and errors had been a problematic to the learning of calculus, and thereby resulting to a poor understanding of the process of teaching and learning among Grade 12 students. Although, several studies had argued that the misconceptions and the difficult nature of calculus experienced among the learners is a thing of great concern, but some of them failed to point out the misconceptions and the errors committed by the students when learning differentiation rules as a subtopics in calculus, and this had created a vacuum to the process of learning differentiation rule in calculus and the mathematics research in general (Chigede, 2016; Jameson et al., 2024). And this had led to students' memorization and route learning procedures which could have a short-term success without a solid conceptual foundation on differential calculus (Oktaç et al., 2019). It is on this ground that one could argued that the gap created in research circle had affected the process of teaching learning of differentiation rule as a topic which has resulted to a low performance of grade 12 students in mathematics as subject in their final.

According to some researchers, the causes of misconceptions and errors among the students when learning differentiation rules could be because of the lack of prior Knowledge on the topics like logarithmic laws, proportionality, integration of concepts and lack of the foundational understanding on some complex differentiation rules among many others (Chikwanha et al., 2022). Going by the gap observed above, the researchers had found it deemed to investigate the misconceptions and errors among grade 12 students when learning differentiation rules. The objective of this study is to identify the misconceptions and errors committed by Grade 12 students, and to suggest the possible means of resolving it to increase the grade 12 students' understanding in differentiation rules and to reduce the experiences of a mass failure encountered among the Grade 12 students during their final school examination in South Africa.

It is on this note that some researchers further argued that the lack of conceptual misunderstandings could cause a form of struggle among the students when it comes to recognize the relationships between variables or the understanding of the underlying principles (Chikwanha et al., 2022). All these challenges as mentioned above could be resolved by identifying the correct variables, applying the chain rule, and simplifying expressions that was found to be problematic (Kandeel, 2021). However, research by Chigede (2016) on advanced level high school learners' misconceptions in differentiation suggest that strong research should concentration of how to improve students' ability to use the different rules and techniques of differentiation, such as the power rule, product rule, quotient rule and other rules. He also reveals that some students have problems in distinguishing a power function from an exponential function, and this implies they had a

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misconception on when to use the power rule to find derivatives, therefore, more efforts should be given to the teaching of differentiation. More so, when learning differentiation, such that f(x) =x3 as a power function, a power rule can be employed to find the derivative, it is not appropriate in f(x) = 3x which is an exponential function.

Chigede further revealed that some students struggled with the application of the product and quotient rules when surds and fractional exponents were involved. And these misconceptions had resulted from a shallow understanding of surds, and rational exponents that should have been mastered during algebra lessons (Cline et al, 2020; Chigede, 2016). To address the gap, the underlisted research question was being addressed in this study:

What are the errors and misconceptions of Grade 12 learners when learning differentiation rules in calculus?

Misconceptions Versus Realities when learning Differentiation rules

There are different forms of misconceptions reported among grade 12 students when learning differential and derivative rules, and these had posted some challenges to the easily recognition of reality and misconception itself when learning calculus. And some of these misconceptions and realities arose at the following stages of learning: One, when equating derivatives to function values at a point- At this point a form of misconception arise when students believe that the derivative of a function at a specific point is equal to the function value at that point. But the reality is that the derivative represents the rate of change of the function at a point, not the actual function value (Stewart, 2015). Two, when a student confused the tangent equation with the derivative function, a form of misconception arose and because of this, some students sometimes think that the tangent equation (the equation of the tangent line) is the same as the derivative function. But the Reality is that the derivative function provides the slope of the tangent line, but they are distinct concepts (Larson & Edwards, 2013).

Three, an argument happens when equating the derivative at a point with the Tangent equation value, and this causes misconception which allows the students to mistakenly believe that the derivative at a point is equal to the value of the tangent equation at that point. While the Reality is that the derivative at a point gives the slope of the tangent line, while the value of the tangent equation represents the function value at that point. Some of these misconceptions could hinder students' understanding of differentiation/derivative when learning differentiation in calculus (Anton et al., 2012).

Another misconception reports that when learning differentiation, it takes too long time to resolve the problem, and because of this it becomes complicated, due to students' beliefs on the learning of derivatives using the definition of the derivative (limit-based approach) as the only way learning, which could be time-consuming and challenging. However, differentiation rules (such as the constant rule, power rule, sum and difference rules, product rule, and quotient rule) allow us to bypass this process and find derivatives more efficiently (Strang, 2020).

More so, there is this misunderstanding about the constant rule which argued that the constant rule is the derivative of a constant function which is zero, and because of this, some students mistakenly think that any function with a

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constant term (e.g., f(x)=8 has a derivative of zero. However, this rule specifically applies to constant functions (horizontal lines), not functions with other terms (Strang, 2020). In addition to this, another misconception is the overgeneralization of power rule, which confirmed that the power rule is powerful for finding derivatives of functions like f(x) = xn , because of this, students may incorrectly apply it to all functions without considering the exponent's restrictions. For instance, they might forget to adjust the exponent when differentiating functions with negative exponents or fractional exponents (Othman et al., 2018). Furthermore, not recognizing the chain rule and the form of misconception which emphasized on the essential for differentiating composite functions which may sometimes overlook the need to apply the chain rule when differentiating nested functions remains issues when learning differentiation rule. And this might mistakenly treat the inner function as a constant, leading to incorrect results (Strang, 2020).

When learning a differentiation rules among grade 12 students, there are some realities to consider, and these include one, student diversity, which recognizes that the students have varying backgrounds, readiness levels, language abilities, learning preferences, and interests (Aliyeva, 2021). Another reality to be considered when learning differentiation is a purposeful choice, which argue that the teaching of a differentiation should involves the deliberate and thoughtful decisions by teachers, and these choices include selecting appropriate instructional approaches, materials, and goals. Therefore, the teachers involved must analyse student achievement, progress, and the instructional needs (Van Geel, et al., 2019). In addition, considering the equity and socio justice needs when learning deafferenting an equation should be aligns with the principles of inclusive education, and this prioritizes the equity and social justice by ensuring meaningful participation of the students and their academic performance regardless of the academic needs (Aliyeva, 2021). It is on this ground that one could say that the teachers teaching differentiation could avoid misconceptions when they assign tasks of varying complexity to different groups of students when learning differentiation rules (Aliyeva, 2021). In summary, the learning of the differentiation rules requires a deep understanding of student needs, purposeful planning, and a commitment. By embracing these realities, educators can create more effective and equitable learning experiences for all students at different levels with grade 12 level inclusive (Aliyeva, 2021; Tomlinson, 2017; Van Geel et at., 2019).

Furthermore, the research conducted by Chigede (2016) on the learners' misconceptions in differentiation focuses on students' ability to use the different rules and techniques of differentiation, for instance, the power rule, product rule, quotient rule and other techniques. His research reveals that students have problems in the distinguishing a power function from an exponential function, this implies that they have some misconceptions on when to use the power rule to find derivatives. While f(x)=x3 is a power function and the power rule can be employed to find the derivative, it is not appropriate in , f(x)= 3x which is an exponential function. His studies also revealed that students struggled with applying the product and quotient rules when surds and fractional exponents were involved. The misconceptions resulted from a shallow understanding of surds and rational exponents that should have been mastered during algebra lessons (Chigede, 2016).

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The research attempted to address the question of what misconceptions Grade 12 learners had in relation to the application of the power rule and finding derivatives of products and quotients of functions involving rational exponents.

Research conducted by Mkhatshwa (2016) on students' reasoning about calculus problems revealed that students had a misconception of the concept of a function where the function is viewed as a set of isolated points. It was also reported that students also had difficulties in understanding the concept of a point of inflection and at times confused critical points with points of inflection. This study further reveals that students has misconceptions on the necessary and sufficient conditions for a point of inflection. This gives a misconception on the point of inflection by showing a flawed understanding of the derivative concept and how it can be applied both to the solving of problems relating to graphs of cubic functions and to addressing real life problems in economics and other contexts (Mkhatshwa, 2016).

Further research by Orton (1983) revealed that students had difficulty in interpreting negative and zero instantaneous rates of change, and this misconception resulted from insufficient knowledge of the derivative concept. Similarly, studies by Tall and Watson (2013) revealed that students had difficulties with utilising visual considerations in resolving calculus problems. The students struggled with sketching graphs of gradients or derivatives of functions given graphically or symbolically, which was evidence of a poor understanding of the derivative concept (Tall, & Watson, 2013). Therefore, this research aimed at assessing the depth of understanding of the derivative concept in Grade 12 students in terms of their ability to apply the knowledge on derivatives to resolve problems involving cubic functions and their graphs.

The research by Ferrini-Mundy and Graham (1994) on student teachers' understandings of concepts and foundations of fundamental calculus, established that these were misunderstand by most learners when learning calculus. A similar observation was made by Porter and Masingila (2000) on university students' studying calculus confirms that a group of students possess shallow understanding of basic calculus which was reported as a fault coming from the lecturers. According to a study reported by Bakri et al. (2021) that established that the sketching of calculus related graphs in mathematical functions was a challenging task for some students. Therefore, students are found struggling with the comprehension and the interrelationship between the algebraic, symbolic, and graphic representation of functions under the application of calculus. It is on this ground that Dlamini et al. (2017) reported that the possible causes of poor performance of high school students in differential calculus established that learners faced challenges in understanding the geometric meaning of the derivative of cubic and quadratic functions. The learners had difficulties understanding that the derivative of a cubic graph gives a parabola, and the derivative of quadratic function gives a straight line.

Other studies also revealed that students' difficulties in solving calculus related problems is an indicator of a misunderstanding of the second derivative's geometric meaning and how the second derivative is related to the first derivative. This is also coupled with the lack of understanding of the implications of continuity

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on differentiability and interpreting the derivative when learning (Barker et al., 2007).

Conceptual framework

This section presents a framework adopted from a recent work of Jameson and others which reports some areas of errors committed by some grade 12 students, and all these errors are reported below in a diagrammatic illustration in figure 1.

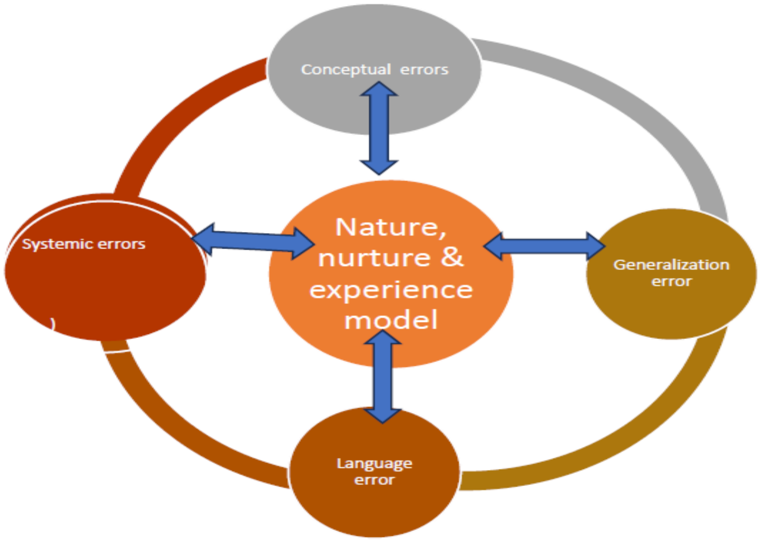

Figure 1: Nature, Nurture & Maturity Dependance Model (NNMDM) (Jameson et al., 2024).

During the learning of differentiation rule in mathematics, this study and available literature had confirmed that there are some errors that comes up from the different aspects of learning it. And these include conceptual error, general error, language error and systemic error. These errors committed by grade 12 students when learning calculus could be basically hanged on the nature, nurture, and experience model as reported by (Jameson et al., 2024). In achieving the goal of this study, the researchers adopted a model applicable to the misconception and errors committed by mathematics students when learning calculus. And this model incorporated the nature of the students' learning calculus as a topic, and lastly the initial experiences gathered when learning the prerequisite mathematics such as logarithm and so on.

Gathering from the data and the result of the finding, it is confirmed that when learning differential calculus, the result from this study has reported in the analysis section had argued that there are misconceptions which arose as a result of low understanding of the previous mathematics concepts like symbol and laws of logarithms. Therefore, one could say that some factors like the nature of the students which is relating to the inherited talent from the parents or others remains important just because it has a positive or negative impact on students' ability to do and calculate mathematics. In addition to this, it could also be confirmed that when learning differentiation in calculus, the training received as students remain important, in fact once could argue that the nature, length and the understanding of the teacher teaching calculus in a high school matter and it may positively or negatively affect the level of understanding of the students. Finally, the experience

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of the students in prerequisite mathematics topics related to calculus remain important. For instant, students with a strong understanding in logarithm could easily understand the differentiation rule, which remain a major part of calculus.

From the explanation of the above-mentioned concepts in the model, one could argue that when teaching differential rules, the adoption of the above listed areas may be of assistance, and theses area; conceptual area, systemics area that deals with the stages of calculation, language aspects, generalization area, and nature of the students, nurture of the students, and the experience of the students remain importance. All these go in line with a view from the data collected and analyzed in the discussion section below in section 4 and 5 below.

RESEARCH METHOD

This section of the study presents the stages adopted before, during, and after the process of data collection, and these stages include research design, participants involved in the study, process of data collection and analysis as well as the ethical consideration for the study. All these were done to increase the accuracy of the study.

Research design

Research design is the stage-by-stage process involved in data collection, and the analysis, to achieve the objectives of the study. It also outlines how to get the relevant information available for the study (Poth & Creswell, 2018). On this note, the study was carried out using a purposive and convenient sampling, and it was done by selecting the Grade 12 mathematics learners at one of the high schools at Limpopo province. This was done because one of the researchers was an educator at the institution, and it was easy and convenient to reach the students. More so, to achieve the aim of this study, a mixed method approach was used during the process data collection by distributing a questionnaire to the students and one-on-one interview section to allow a rich data.

Participants

Before the process of data collection the researchers carefully select the appropriate group that were involved in the learning of mathematics among grade 12 students at a high in Limpopo. Therefore, 35 grade 12 students were selected and a survey questionnaire was employed by the researchers to locate the ideas of the students on differentiation rule, after the process of collection of the questionnaires, 7 study were selected due their ideas on differentiation rule in calculus, while 5 students were selected using their worksheet reported and submitted to the researchers. These 5 students were interviewed and reported in the section below.

Data collection and analysis process

For data collection process, one of the researchers was involved in the process because he was an educator at the institution, and it was easy and convenient to reach the students. Moreso, he understands the details of the grade 12 mathematics curriculum on the differential calculus, differentiation, and integration. With this approach, it was an easy task to interact with the students after the ethical clearance has been obtained. More so, after data collection process, the

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process of sorting and coding the collected data was also done to allow anonymity. Hence, the whole data collected were interpreted thematically to allow better understanding of the reader. hence the selected sample provided the required information (Cohen et al., 2017).

Ethical Considerations

To confirm the ordinality of the study, the researchers presented a written consent to the students, which was signed by the students to endorse their consent before venturing into the process of data collection.

RESULTS AND DISCUSSION

The result of the study on the misconceptions and errors among grade 12 Students when learning differentiation rules was reported using a sample question taken from grade 12 curriculum in South Africa. This question was adopted to analyse the data and report how students apply their mathematics knowledge in differentiation rule and the misconceptions attached to it. To properly analysed this, some data were collected from the grade 12 students learning differentiation rules in a calculus related course. Furthermore, after the process of data collection, a process of categorization was done by the researchers by arranging the result of the findings in line with the similar view as gathered from the participants. Creswell and Poth (2018) empathetically report that a data categorization could be done thematically in order to discuss the similarities observed from the outcome of the findings. Therefore, introducing a coding and categorization method could be of assistance to the researchers in sorting and describing views of the participants (Maxwell, 2008; Watkins & Gioia, 2015). On this note, the researchers adopt the result of the survey questionnaire which led to the selection of 7 grade 12 students learning differentiation rules out of 35 students that participated in the study. This implies that out of 100% of the participants, the result indicates that in survey question (1i) about 31% of the students supplied incorrect answers due to some misconception, while in survey question 1ii about 63% of the students got some incorrect answers. This implies that there is a high percentage of misconception which led to a high rate of error among grade 12 students when learning differential rules.

Table 1: Analysis of the survey results per test item in percentages

Test item	No. of correct answers	% of correct answers	No. of partially correct answers	% of partially correct answers	No. of incorrect answers	% of incorrect answers
1 i	15	43	9	26	11	31
1ii	2	6	11	31	22	63

From the table 1 shown above, it is significant to note that the total 94% , with a breakdown of 31% of students had an incomplete answers in question 1i while 63% of students in the question 1ii had incomplete answers with different forms of errors and misconceptions which lead to a poor performance of the in the differentiation rules. To probe into the sources and causes of misconceptions and errors among grade 12 students when learning differentiation rules, the researchers

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further select 7 students among the grade 12 students having an adequate knowledge on differentiation, after which 5 were interviewed to adequately understanding of their misconception when learning differentiation rules. For the accuracy of the study, a research question (RQ) displaced below was used as a guide:

What are the errors and misconceptions of Grade 12 learners when learning differentiation rules in calculus?

In answering this RQ, some categories were stated and adopted in line with themes as reported below.

Misconceptions on rules for differentiation/derivatives among grade 12 students

In getting the misconceptions and errors committed by grade 12 students in Limpopo, the sample question stated below was served as a tool in determining students' level of understanding and misconceptions when learning differentiation rules. After the test using the sample question, five students were selected for an interview which include, L8, L9, L10, L11 and L12. This was done due to their understanding and views on differentiation rules. And the details of their views are hereby displaced and analysed below.

Sample question 1. *Given the following problem: Differentiate.*

(i) $f(x) = (x^3+1)(x-5)$

(ii) $y = \frac{\sqrt{x} - 4}{\sqrt{x}}$

(a) Identify and explain the errors in the following solutions

Learner A	**Learner B**
(i) $f(x) = (x^3 + 1)(x - 5)$	(ii) $y = \frac{\sqrt{x} - 4}{\sqrt{x}}$
$f^{/}(x) = (3x^2)(1)$	$\frac{dy}{dx} = \frac{x^{\frac{1}{2}-4}}{x^{\frac{1}{2}}}$
$= 3x^2$	$= \frac{\frac{1}{2}x^{\frac{-1}{2}}}{\frac{1}{2}x^{\frac{-1}{2}}}$
	$=1$

Write the correct solutions for both (i) and (ii)

Figure 2: Sample question on students test on the application on differentiation rule

Sample question 1 above, was intended to check on the learners' procedural knowledge of differentiation using the rule that if $f(x) = ax^n$, then $f^{/}(x) = anx^{n-1}$. The question also tested the learners' conceptual knowledge of surds and the laws of exponents, as well as their algebraic skills of simplifying fractions, surds and exponents by finding derivatives of (i) $f(x) = (x^3 + 1)(x - 5)$ and (ii) $y = \frac{\sqrt{x} - 4}{\sqrt{x}}$. In resolving this problem, some learners in this group of nineteen

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learners, (19), found derivatives of each bracket and then multiplied to get the final answer as reflected in Figure 2.1 by L24.

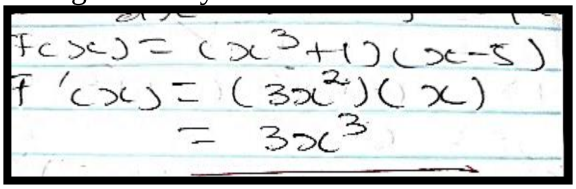

Figure 2.1: L24's solution to question 1b(i)

The result of the participants coded L24 shows that there is a generalization or transfer error, as shown in the equation $\frac{d}{dx}f(x)g(x) \neq \frac{d}{dx}f(x)\frac{d}{dx}g(x)$. This learner, *L24*, found the derivative of each bracket separately first but committed yet another error on derivative of (x-5) which was given as x instead of 1. Strategies learnt earlier are overgeneralized and applied in calculus where they do not apply, and this was compounded by a weak foundation on rules of exponents.

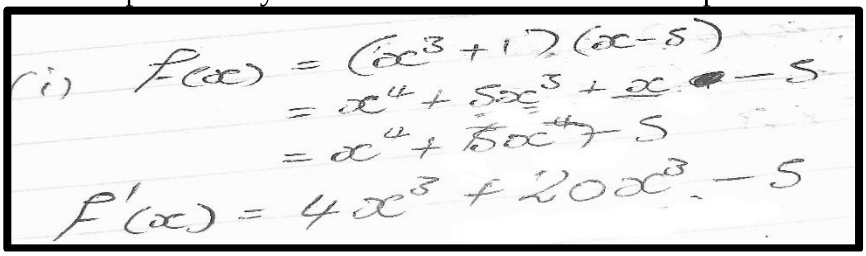

Figure 2.2: L29's solution to question 1b(i)

Similarly, the answer above shows that the learners coded L29, failed to expand $(x^3+1)(x-5)$ or failed to simplify the resultant algebraic expression as reflected in the work of L29 in figure 2.2 above. This sample in figure 2.2 demonstrates the impact of a weak foundation on extrinsic calculus concepts like algebraic skills on the learning of other concepts such as derivatives. This learner, L29, had problems applying the general rule for derivatives which state that when g*iven* $\boldsymbol{f(x) = ax^n}$, then $\boldsymbol{f'(x) = anx^{n-1}}$, because of a poor grasp of algebraic skills where unlike terms are wrongly grouped, and addition is replaced by multiplication. While $f(x) = x^4 - 5x^3 + x - 5$ was correct, the derivative $f'(x) = 4x^3 + 20x^3 - 5$ was incorrect as a result flaws in the learner's algebraic skills. One major challenge exposed by this question was learners' weak grasp of laws of exponents. Furthermore, it was also confirmed that some students also struggle with the application of laws of exponents and surds. Exponents and surds are extrinsic calculus concepts which are required as prior knowledge for the successful learning of new concepts in calculus. Some learners in this group gave the following solutions:

From $f(x) = x^4 - 5x^3 + x - 5$, the derivative was given as $f'(x) = 4x^3 - 15x^2 + x$ or $f'(x) = 4x^3 - 15x$. Others simplified y= $\frac{\sqrt{x}-4}{\sqrt{x}}$ to y =$\frac{x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}$ = $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{4}{x^{\frac{1}{2}}} = x - 4x^{-\frac{1}{2}}$ *or y=* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = 0 - 4x^{-\frac{1}{2}}$; as confirmed by the following four samples

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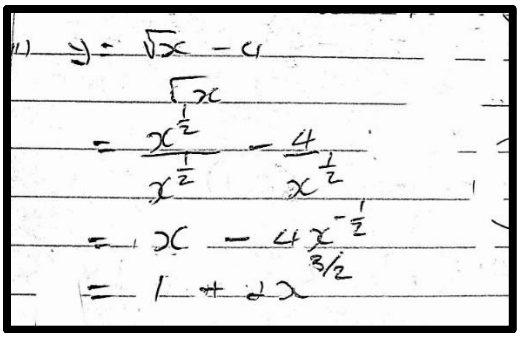


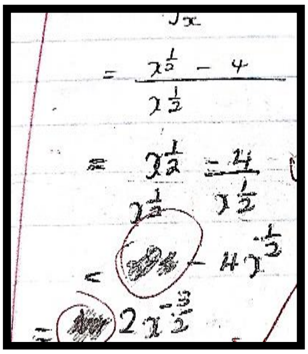
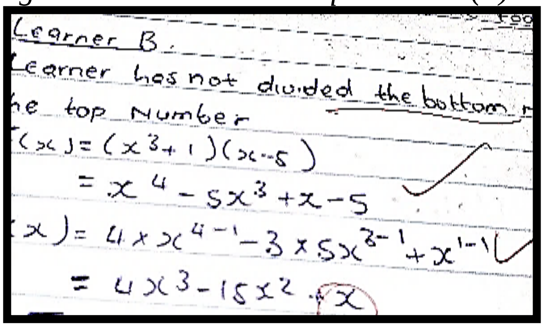


Figure 2.3-L8 solution to question 1b(ii) *Figure 2.4-L9 solution to question 1b(ii)*



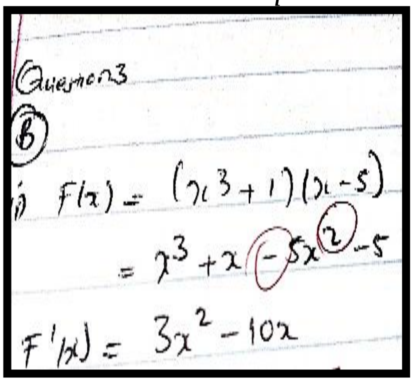


Figure 2.5-L2's solution to question 1b(i) *Figure 2.6-L23's solution to question*

From the above samples in figure 2.3 $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ = *x* for *L8*, and in figure 2.4, $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ = 0 for *L9*; and the derivative of $f(x) = x^4 - 5x^3 + x - 5$ was given as $f^{/}(x) = 4x^3 - 15x^2 + x$ in Figure 2.5 for L2 , and the derivative of $f(x) = x^3 + x - 5x^2 - 5$ was given as $f^{/}(x) = 3x^2 - 10x$ in figure 2.6 for L23. All the four samples in Figure 2.3, 2.4, 2.5, and 2.6, confirm the existence of a misconception on the derivative of *x* or the value of x^0 where wrong answers of $x^0=x$ and $x^0=0$ were often given instead of the correct answer of $x^0=1$.This misconception is a result of flaws in the learners' knowledge of laws of exponents which impact negatively on the learners' efforts to acquire new mathematical knowledge on calculus. This goes in line with cline who warns against having a poor understanding of mathematics rule, which could lead to students having a conceptual errors and generalization, which was committed by L8, L9, L2 and L28 (Cline et al., 2020).

The views of other participants on rules for derivatives after interview section

After the result of the worksheet supplied by grade 12 students on their misconceptions and views on the rules of derivatives, a group of five learners *L8, L9,L10,L11* and *L12,* were requested to clarify their written responses to the question 1 due to their understanding on derivatives, and the result of the interview is hereby reported below on individual basis.

Learner 8's misconceptions on rules of derivatives after an interview section

The views of the participant coded L8 as indicated and reported in the worksheet below indicates some errors and misconception as shown in the answers below. In the sample of the question displaced above, one of the researchers that

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interview the participants further probed L8 using an interview guide to further understand his views on the participants on the misconception and errors encountered when learning the differentiation rules. And the result of the interview section was hereby reported as follows with the solution to sample question below. The answer to the question 1 shows by the worksheet indicates that.

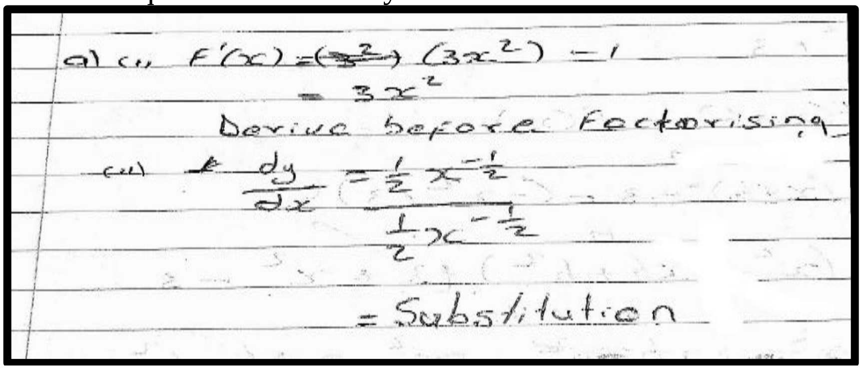

Figure 2.7-L8's solution to question 1 (i)

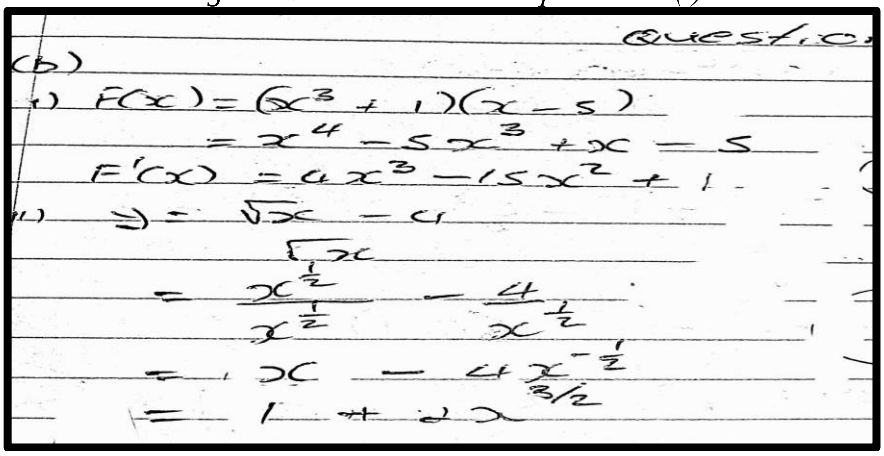

Figure 2.8-L8's solution to question 1(ii)

In getting a better understanding of the answers reported above, one of the researchers further probed the students (L8) with some interview questions which are reported below.

Researcher: *Can you please explain what you mean by 'derive before factorizing' in 3(a)?*

L8 : *Learner A is finding derivative of each bracket without removing the factors first. Learner B should not substitute when there is a fraction from* $\frac{x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}$ *to* $\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}x^{-\frac{1}{2}}}$.

Researcher : *Is learner B not finding derivatives?*

L8 : *Learner B must not put the derivatives in a fraction, he must remove fraction first.*

Researcher : *In your answer you wrote* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{4}{x^{\frac{1}{2}}} = x - 4x^{-\frac{1}{2}}$, *how did you get x from* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$?

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L8	:	*I cancel the exponents because they are equal, so* $\frac{1}{2} \div \frac{1}{2}$ *= 1 and we remain with x.*
Researcher	:	*What is your answer to* $\frac{2^2}{2^2}$ *?*
L8	:	$\frac{2^2}{2^2}$ *=4/4=1 because the numbers are equal, I can get it from calculator.*

The extract of students coded L8 shows that he demonstrates some understanding of the rules of derivatives for products and quotients, shown that $\frac{d}{dx}f(x)g(x) \neq \frac{d}{dx}f(x)\frac{d}{dx}g(x)$ and $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) \neq \frac{\frac{d}{dx}(f(x))}{\frac{d}{dx}(g(x))}$. However, the learner had a flawed understanding of the laws of exponents where, for him, $x^0=x$ instead of $x^0 =1$, and this misconception impacted negatively on his resolution of calculus questions. The second misconception in L8's solution is where $y=f(x) =f^{/}(x)$, in this statement $y =x -4x^{\frac{-1}{2}} = 1+2x^{-\frac{3}{2}}$. This is an incorrect statement as $f(x) \neq f^{/}(x)$. From the views of the participants coded L8, the researchers could clearly argue that L8 had committed an error related to the mathematics rule because of this, a systemic error was committed which affect the result of the problem, thereby causing forms of misconceptions when applying differentiation rules. This goes in line with Cline and other researchers views on errors and misconception reported by students in mathematics class (Cline et al., 2020).

***Learner 9's misconceptions on rules of derivatives* after an interview section**

The view participant coded L9 as shown in the solution to the sample question stated below indicates some form of errors and misconception as reported in the answers below. Gathering from the sampled question displaced above, one of the researchers probed L9 using an interview guide to further understand the views of the participants on the misconception and errors encountered when learning the differentiation rule. And the results of their views are hereby reported below.

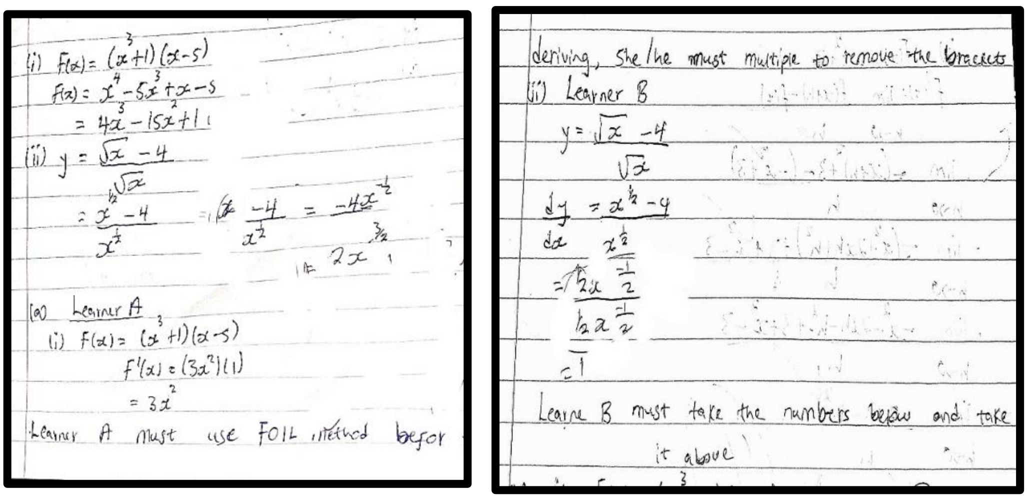

Figure 2.9: L9's written responses to question 1

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In getting a better understanding of the answers reported above, one of the researchers further probed the second grade 12 students coded L9 with some interview questions which are reported below with his responses.

Researcher	:	*When you wrote that 'learner B must take numbers below and take it above, which number are you referring to?*
L9	:	*Learner B must remove* $x^{\frac{1}{2}}$ *below so there is no fraction.*
Researcher	:	*When you wrote that "learner B must take numbers below and take it above", which number are you referring to?*
L9	:	*Learner B must remove* $x^{\frac{1}{2}}$ *below so there is no fraction.*
Researcher	:	*In your answer you wrote* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{4}{x^{\frac{1}{2}}} = -4x^{-\frac{1}{2}}$, *how did you get nothing or zero from* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$*?*
L9	:	*When you subtract exponents* $\frac{1}{2} - \frac{1}{2} = 0$ *, from* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ *final answer is 0.*
Researcher	:	*What is your answer to* $\frac{2^2}{2^2}$ *?*
L9	:	*The answer is 1 , you are dividing number by itself ,* $\frac{2x2}{2x2}$ *=1.*

Gathering the view of participants coded L9, the result demonstrates that L9 understood the rule that $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) \neq \frac{\frac{d}{dx}(f(x))}{\frac{d}{dx}(g(x))}$, but had a flawed understanding of laws of exponents. From her explanation that $x^{\frac{1}{2}}$ must be "*taken above",* her solution was expected to show $\frac{x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}$ =($x^{\frac{1}{2}}$ -4)$x^{-\frac{1}{2}}$ followed by the appropriate simplification using laws of exponents. The error in the solution for the derivative is a result of a weak foundation on laws of exponents where $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = 0$ instead of $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = x^0$ =1. However, when presented with a numerical fraction $\frac{2^2}{2^2}$, L9 got the correct answer. This shows L9 has a chance of committing a system error which may occur along the way due to her poor understanding on a law of exponential rule. On this ground one could argue that the participant coded L9 commit a systemic error which may generally affect the stages of learning differentiation rule and final resulted a misconception. This goes in line with the views of some researchers who argue that a systemics errors is a form of errors that arose as a result students not having adequate knowledge of the differentiation rule, which if not addressed could result to the errors that may lead to misconception (Cline et al., 2020; Jameson, et al, 2023; 2024).

Learner 10's misconceptions on rules of derivatives after an interview section

The view participant coded L10 as indicated and reported as the solution to the problem on the worksheet indicates that there are some errors and

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misconceptions as shown in the answers below. Gathering from the answer to the sample question adopted, one of the researchers probed L10 using an interview guide to further understand the views of the participants on the misconception and errors encountered when learning the differentiation rule. And the results of their views are hereby reported below.

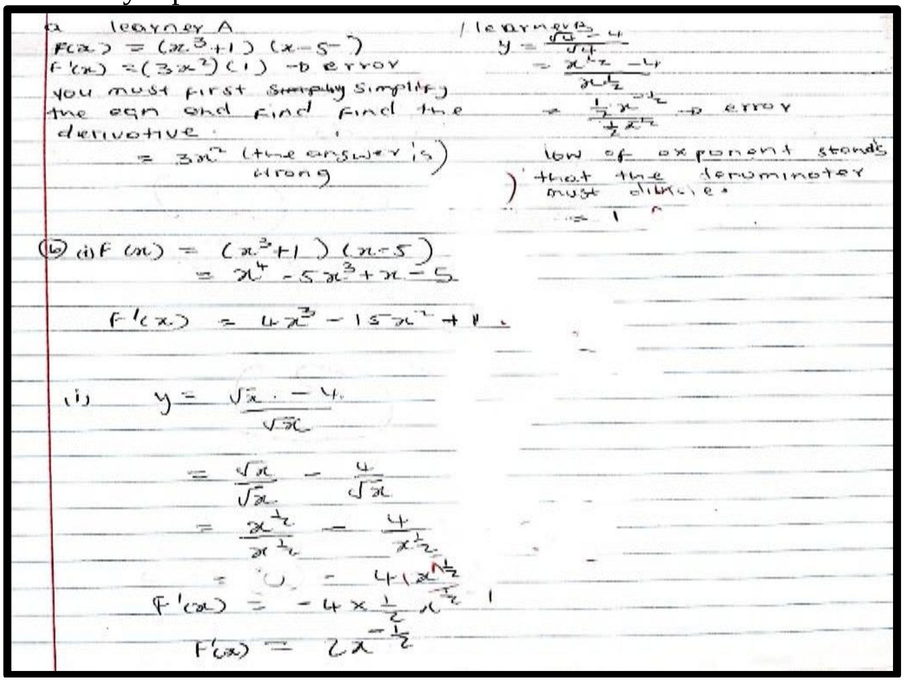

Figure 2.10: L10's written responses to question 3

In getting a better understanding of the answers reported above, one of the researchers further probed the students (L10) with some interview questions which were reported below.

Researcher : *In your answer, you wrote* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{4}{x^{\frac{1}{2}}} = 0 - 4x^{\frac{1}{2}}$*, how did you get zero from* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ *and* $4x^{\frac{1}{2}}$ *from* $\frac{4}{x^{\frac{1}{2}}}$*?*

L10 : *I used laws of exponents which says subtract exponents when you divide,* $\frac{1}{2} - \frac{1}{2} = 0$ *, so I wrote 0 to simplify.*
$\frac{4}{x^{\frac{1}{2}}} = 4x^{\frac{1}{2}}$ *, the numerator does not have x, we take denominator to the top so that we can use rule for finding derivative from exponents, rule says if* $f(x)=ax^{n}$ *then* $f'(x)=anx^{n-1}$

The participants coded L10's misconception on the derivatives rules as indicated by his worksheet was because of a flawed understanding of laws of exponents, specifically the law which states that $x^{0}= 1$. From $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = x^{\frac{1}{2}-\frac{1}{2}} = x^{0} = 1$, L10 did not appreciate he must simplify exponents of x and that $x^{0} \neq 0$. The second

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error of $\frac{4}{x^{\frac{1}{2}}} = 4x^{\frac{1}{2}}$ is also a result of the learner's failure to realiSe that $\frac{4}{x^{\frac{1}{2}}} = \frac{4x^0}{x^{\frac{1}{2}}} = 4x^{0-\frac{1}{2}} = 4x^{-\frac{1}{2}}$. Therefore, a weak background knowledge of the laws of exponents impacted negatively on the learner's progress in understanding new calculus concepts. This resulted to the student committing a generalization error or transfer error which could resulted to a misconception of the students learning a differentiation rule, as well as an exponent rule. Finding goes in line with Makoye and Luneta who reported that the poor management of the application of the differentiation rules may result to the form of errors known as a systemic and generalization which could eventually result into students committing forms of misconception (Cline et al., 2020).

Learner 11's misconceptions on rules of derivatives after an interview section

The view participant coded L11 as indicated and reported in the worksheet on the solution to the problem indicates some errors and misconceptions as shown in the answers below. Gathering from the sample question displaced, one of the researchers probed L11 using an interview guide to further strengthening the views of the participants on the misconceptions and errors encountered when learning the differentiation rule. And the results of their views were hereby reported below.

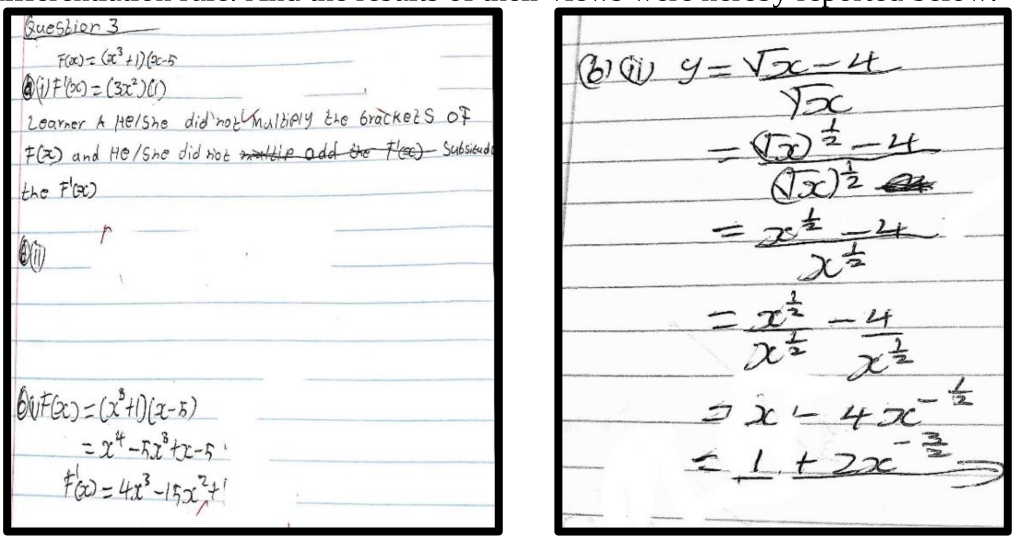

Figure 2.11: L1's written responses to question 3

In getting a better understanding of the answers reported above, one of the researchers further probed the students (L11) with some interview questions which are reported below.

Researcher	:	*Explain what you mean by "did not substitute the $f^{/}(x)$"*
L11	:	*$f^{/}(x)=(3x^2)(1)$ is only derivative of x^3, he must remove brackets to get the correct derivative.*
Researcher	:	*You wrote $\frac{(\sqrt{x})^{\frac{1}{2}}-4}{(\sqrt{x})^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}$, explain why.*
L11	:	*Where there $\sqrt{x}$, it means 'square root of x' and using exponent.*

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Researcher : *In your answer you wrote* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{4}{x^{\frac{1}{2}}} = x - 4x^{-\frac{1}{2}}$*, how did you get x from* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$*?*

L11 : *The law of exponents for division says we must subtract exponents, and* $\frac{1}{2} - \frac{1}{2} = 0$ *, so I remain with x.*

Researcher : *Simplify* $\frac{3^2}{3^2}$

L11 : $\frac{3^2}{3^2} = \frac{9}{9} = 1$

The participant coded L11 misconceptions on the learning of the derivatives rules emanates from a poor mastery of laws of exponents and surds. Firstly, $\left(\sqrt{x}\right)^{\frac{1}{2}} = \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}}$ which is not what the learner wanted here when he wrote $\frac{(\sqrt{x})^{\frac{1}{2}}-4}{(\sqrt{x})^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}$. The intention was to remove $\sqrt{\square}$ and replace it by exponent 1/2, but writing both at the same time changes the mathematical statement and $\frac{(\sqrt{x})^{\frac{1}{2}}-4}{(\sqrt{x})^{\frac{1}{2}}} \neq \frac{x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}$. The second misconception is about x^0, which, *L11* equates to *x*, yet $x^0 = 1$ so that $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = 1$. While the learner can deal with $\frac{3^2}{3^2} = \frac{9}{9} = 1$, he failed to generalize to $3^{2-2} = 3^0 = 1$. The student's progresses in acquiring new knowledge and resolving problems in differentiation rules is negatively affected by earlier misconceptions developed in learning concepts in surds and exponents. This implies that student coded L11 has committed some errors such as hypothesis error and systemic error which came in because of not having enough knowledge on surd which affected the easy resolution of the given problem (Bakri, 2021; Green et al., 2008; Makonye & Lunata, 2012).

Learner 12's misconceptions on rules of derivatives after an interview section

The view participant coded L12 as indicated and reported in the worksheet on the solution to the problem indicates some errors and misconception as shown in the answers below. Gathering from the sample question displaced above, one of the researchers probed L12 using an interview guide to further understand the views of the participants on the misconception and errors encountered when learning the differentiation rule. And the results of their views were hereby reported below.

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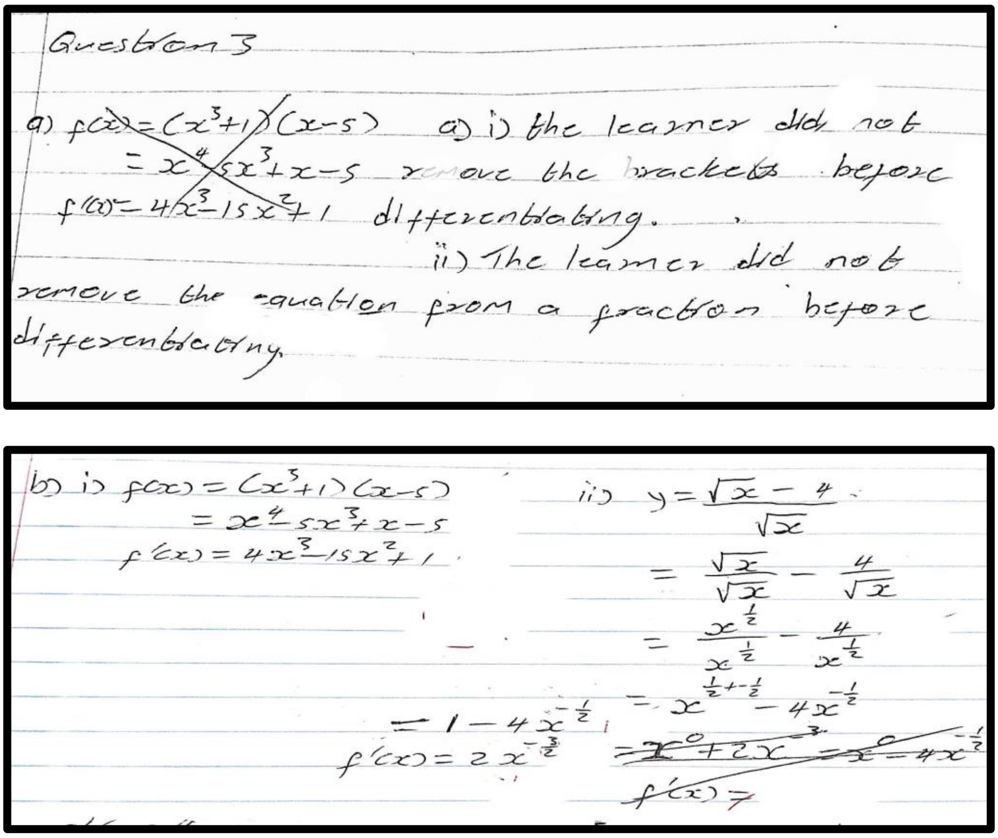

Figure 2.12: L12's written responses to question 1

In getting a better understanding of the answers reported above, one of the researchers further probed the students (L12) with some interview question which was reported below.

Researcher	:	*When you were identifying errors in Learner B's working, you mention that he "did not remove the equation from fraction", explain what you mean.*
L12	:	*He should remove x from the bottom of this fraction* $\frac{x^{\frac{1}{2}}-4}{x^{\frac{1}{2}}}$ *before finding derivative. He must divide each term at the top by* $x^{\frac{1}{2}}$*.*

Despite the challenges encounter by other participants, the student coded L12 has mastery of the mathematical rules governing the determination of derivatives of products and quotients of functions, i.e. $\frac{d}{dx}f(x)g(x) \neq \frac{d}{dx}f(x)\frac{d}{dx}g(x)$ and $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) \neq \frac{\frac{d}{dx}(f(x))}{\frac{d}{dx}(g(x))}$. The learner's reference to "equation" in his explanation appears to be a grammatical error as opposed to a mathematical misconception. Four out of the five interviewed learners, *L8*, *L9*, *L10* and *L11*, have been struggling with the question of whether $\boldsymbol{x}^{\frac{1}{2}-\frac{1}{2}} = \boldsymbol{x}$, or $\boldsymbol{x}^{\frac{1}{2}-\frac{1}{2}} = 0$, both of which are incorrect, as the correct answer is $\boldsymbol{x}^{\frac{1}{2}-\frac{1}{2}} = x^0 = 1$. While the learners can give the

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correct answer of 1 for $\frac{4}{4}$ or $\frac{3^2}{3^2}$, they fail to generalize the principle to $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$. This is evidence that the students' knowledge of laws of exponents is instrumental as opposed to their relational knowledge. Once learners have doubts on the laws of exponents, then they are likely struggle with mastering calculus concepts where the standard rule for derivatives is defined in terms of exponents, i. e. for $\boldsymbol{f(x) = ax^n}$, $\boldsymbol{f'(x) = anx^{n-1}}$ (Herhelm, 2023). Hence a lack of a strong foundation on working with exponents and surds creates a barrier for the students to deal successfully with differential rule among L8, L9, L10 and L11, which resulted to the students in coming generalization errors.

Discussion

This study was intended to test the students' ability to correctly apply their knowledge of differentiation rules and to resolve some algebraic problems. It also checked on the students' ability to calculate some major aspects of derivatives in line with grade 12 CAPS curriculum. This study also reports common misconceptions that could affect students' abilities to positively transfer their understanding when learning differentiation rules. These misconceptions discovered by the researchers when interacting with the students when teaching and learning differentiation rule are hereby reported in four subheadings as indicated below. And these include.

1. Generalisation or transfer errors and misconception when learning differentiation rule.
2. Wrong hypothesis and its misconception when learning differentiation rule.
3. Systemic errors and its misconception when learning differentiation rule.
4. Language errors and its misconceptions in when learning differentiation rules.

The themes listed above are hereby discussed to produce the result of the finding as reported and interpreted above.

Generalisation or transfer errors and misconception when learning differentiation rule

The findings of this study based on sample question 1 was intended to address the issue of generalisation or transfer errors on finding the derivative of products of functions and quotients of functions. Secondly, the question also intended to assess the algebraic skills of algebraic multiplication and division and the laws of rational exponents. This study shows that some Grade 12 students (L8, L10, and L11), have the misconceptions that the derivative of the product of two functions is equal to the product of the derivatives of the functions. Some learners interpreted the derivative of a quotient of functions to be equal to the quotient of the derivatives of the functions. The other misconception was on the value of x^0 where some learners gave the answer of $x^0=x$ and others wrote $x^0=0$ after failing to apply laws of exponents correctly on $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$. The solution to the sample question 1 of student coded L24 in figure 2.1 finds derivative of each bracket first before multiplying the answers. L29 in Figure 4.19 fails to simplify the algebraic expression for the product of the two brackets. L2 and L23 in Figure 4.20 struggle

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with derivative of x in $f(x) = x^4 - 5x^3 + x - 5$ and for $f\ (x) = x^3 + x – 5\ x^2 - 5$. These samples would justify the conclusion that some misconceptions are as a result of generalisation errors (s2) where L24 erroneously assumes that $\frac{d}{dx}f(x)g(x) = \frac{d}{dx}f(x)\frac{d}{dx}g(x)$, maybe because they have learnt that $\sqrt{xy} = \sqrt{x}\sqrt{y}$. This is what Jameson et al., (2023) that referred to as generalisation or transfer error as $\frac{d}{dx}f(x)g(x) \neq \frac{d}{dx}f(x)\frac{d}{dx}g(x)$. Strategies learnt earlier are overgeneralised and applied in calculus where they do not apply. This goes in line with Cline and others who warn against having a poor understanding of mathematics rule, which could lead to students having a transfer errors and generalization, which was committed by L8, L10 and L11 among many other (Bakri et al., 2021; Cline et al., 2020). Comparatively, the poor understanding of differentiation rules could be linked to the misplacement or misapplication of the stages of learning and doing mathematics. Therefore, emphasizing on the prerequisite mathematics at the previous classes remain important.

Wrong hypothesis and its misconception when learning differentiation rule.

This section of the discussion reports that the research data suggested by some grade 12 students highlight the major role that an in-depth knowledge of laws of exponents plays in enabling learners to deal successfully with calculus concepts are missing among sone students. This was discovered by the researchers gathering from some participants coded, L11 and L28 which resulted to the students struggling with the application of the laws of exponents and surds. These examples in figure 2.8, where L8 writes $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = x - 4x^{-\frac{1}{2}}$; and L9 *writes* $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = 0 - 4x^{-\frac{1}{2}}$, and giving the derivative of $f(x) = x^4 - 5x^3 + x - 5$ as $f^{/}(x) = 4\ x^3 – 15\ x^2 + x$; or $f^{/}(x) = 4\ x^3 – 15\ x^2$, is indicative of serious flaws in learners' understanding of laws of exponents and surds. The students had a misconception that $x^{1-1} = x$, or $x^{1-1} = 0$, yet the correct answer is $x^{1-1} = x^0 = 1$. These are examples of wrong hypotheses used where the learners are using faulty results on the applications of laws of exponents to find derivatives of functions and this affect the result. The knowledge of laws of exponents is critical in applying the standard rule for derivatives which states that for $f(x) = ax^n$, $f^{/}(x) = anx^{n-1}$, which is used extensively in the study of differentiation and calculus. Hence, a lack of a strong foundation on working with exponents and surds creates a barrier for the learner to deal successfully with derivatives concepts. On this note, Bakri et al. (2021) argue that the early acquisition of the essential algebraic knowledge by students could help to create the appropriate foundational base for learning of some mathematical related concepts. In view of this, developing a strong understanding of foundational mathematics aspects related to differentiation rules could not be toyed.

Systemic errors and its misconception when learning differentiation rule

This is a form of error that arose due to mistake from the procedural application of mathematics concepts. And this forms a such of misconception and error among the students when learning differentiation. Gathering from the data collected from grade 12 students, participants coded L8 and L9 were found with

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some procedural mistakes, and this had resulted to a systemic error which had generally affect the stages of learning differentiation rule and final resulted to a misconception. This goes in line with the views of some researchers who argue that a systemics errors is a form of errors that arose as a result students not having adequate knowledge of the rule which if addressed may result the errors that may lead to misconception (Jameson et al., 2023).

Language errors and its misconceptions in when learning differentiation rules

Mathematics is a numeric subject that help the students to interpret the quantitative aspect of physical science and other managerial aspects. But the fact remains that the understanding of some numerical symbol and language remain important when learning some advance level mathematics concepts like differentiation rule. Gathering from the worksheets displaced above, many students investigated lack the understanding between concepts like $f(x)$ *and* $f^{/}(x)$ and this affected some students in applying this to their learning of function and derivative. Eventually some of these concepts among many others affect the learning of differentiation rule and the result. This could be seen in the result sheet and the interview section reported by participants coded L24, L8 and L9 among many others. It is on this ground that researchers argue that this goes in line with Cline and others who argue that the poor understanding of mathematics language could affect the students learning, and lead to misconception when learning (Bakri et al., 2021; Cline et al., 2020). From the views of some participants, the language of learning mathematics remains tools for better understanding of differential rule in a calculus related topics

CONCLUSION

The misconceptions found among the grade 12 students when learning differentiation rules include generalisation or transfer errors, conceptual error, wrong hypothesis used to learn new concepts (Hypothesis error), and systemic errors due to a procedural mistake. Gathering from sample question 1 adopted, it was intended to address the issue of generalisation or transfer errors on finding the derivative of products of functions and quotients of functions. Secondly, the question also intended to assess the algebraic skills of algebraic multiplication and division and the laws of rational exponents.

This study shows that some Grade 12 learners have the misconceptions that the derivative of the product of two functions is equal to the product of the derivatives of the functions. Some students interpreted the derivative of a quotient of functions to be equal to the quotient of the derivatives of the function. All these were because of the poor understanding of procedural required, poor conceptual understanding, and the wrong hypothesis prediction.

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