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Manipulatives on Ghanaian Basic 8 Students' 3d Geometry Achievement

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Abstract

This study examines the impact of manipulatives on Basic 8 students' achievement in three-dimensional (3D) geometry using an experimental research design. A total of 154 students were sampled from a population of 665 using a G*Power software-based estimation, with participants randomly assigned to control (77 students) and experimental (77 students) groups. Data collection involved a pretest-posttest achievement test, questionnaires, and structured interviews. Statistical analysis was conducted using t-tests to compare mean achievement scores between groups, and thematic analysis was applied to qualitative responses. The results indicated that while traditional teaching methods improved students' 3D geometry achievement, the use of manipulatives led to significantly higher gains. Additionally, there was no significant gender-based difference in achievement when manipulatives were used. Questionnaire responses revealed that students developed positive attitudes and perceptions towards 3D geometry when taught with manipulatives, and interview findings suggested an improvement in conceptual understanding. The study concludes that manipulatives are effective tools for enhancing students' comprehension of spatial relationships and recommends their systematic integration into the mathematics curriculum.

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INTRODUCTION

Geometry is a fundamental branch of mathematics that enhances students' spatial reasoning, logical thinking, and problem-solving skills. It plays a critical role in various disciplines, including architecture, engineering, and computer science. In the Ghanaian curriculum, geometry is introduced at the basic level, where students learn about two-dimensional (2D) and three-dimensional (3D) shapes (Ministry of Education. (2020). However, research indicates that students frequently struggle with understanding. Manipulatives enable pupils to perceive and engage with geometric concepts, making abstract notions more apparent. According to Ahmad and Siller (2024), children who use manipulatives develop stronger spatial reasoning and problem-solving skills than those who learn using traditional techniques. Manipulatives give students a physical tool to investigate geometric relationships, which promotes deeper learning and retention. Given the difficulties students have in mastering 3D geometry, incorporating manipulatives into education could help bridge the gap between theoretical learning and practical application.

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Numerous research investigations have demonstrated the efficacy of manipulatives in teaching geometry. For example, Jafri et al. (2017), discovered that employing physical manipulatives like cubes, pyramids, and prisms dramatically increased students' knowledge of volume and surface area concepts. Similarly, Ahmad (2024), found that virtual manipulatives in dynamic geometry software improved students' spatial visualization skills, notably their grasp of transformations and symmetry. Guan et al. (2024), found that students who utilized manipulatives scored better on geometry examinations and were more motivated to learn.

Physical manipulatives are tangible 3D objects such as wooden blocks, plastic models, and geometric solids that enable students to physically manipulate and investigate spatial features. Virtual manipulatives are interactive digital tools that allow students to modify geometric structures on-screen, most commonly utilizing dynamic geometry software such as GeoGebra (New citation required). Some researches propose hybrid techniques, which combine physical and virtual manipulatives to provide an enhanced learning experience (Zhang & Cutler, 2024). Despite the good effects of manipulatives, their use in classrooms is variable. Some teachers lack training in their proper application, while others may not have access to the appropriate materials. This raises the need for structured integration of manipulatives into the mathematics curriculum.

While various studies have looked into the benefits of manipulatives in geometry learning, there has been little research on their impact on Basic 8 students in Ghana. Most studies concentrate on general mathematics accomplishment without examining the impact of manipulatives on 3D geometry topics including spatial reasoning, visualization, and geometric transformations. Furthermore, previous research has not always compared different manipulative kinds to identify which is most successful in specific learning circumstances (Anamuah-Mensah & Mereku,2005; Mensah-Wonkyi, & Adu, 2016). This study will use an experimental strategy to investigate the impact of manipulatives on students' 3D geometry achievement. Unlike prior research, which focused on general learning results, this study investigates how different manipulative forms affect students' conceptual comprehension and attitudes toward geometry. The study contributes to the growing body of knowledge on effective geometry teaching strategies and provides empirical evidence to support the structured integration of manipulatives in Ghanaian classrooms.

Imagery and Spatial Reasoning in Geometry Learning

Spatial thinking is a fundamental cognitive function that allows people to envision, manipulate, and comprehend geometric relationships. It is necessary to understand 3D geometry ideas and solve spatial difficulties (Almulla, 2023). According to Salehi et al. (2023) and Harris et al. (2021), spatial reasoning is divided into three major components:

a) *Spatial Relations*: The ability to sense and mentally modify objects' positions and orientations in space.
b) *Spatial Visualization*: The ability to mentally manipulate, spin, and reorganize geometric shapes.

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c) *Spatial Orientation*: The ability to understand how objects appear from different viewpoints or perspectives.

Van Hiele's (1986) theory of geometric thinking also sheds light on the stages of spatial reasoning development. The five levels, Visualization, Analysis, Abstraction, Deduction, and Rigor, represent the path from identifying shapes to understanding their deeper qualities and relationships. Geometry learning relies heavily on mathematical visualization (Uygun & Güner, 2021). According to Ziatdinov and Valles (2022), visualizing entails generating mental representations of geometric forms to aid in problem solving and intellectual understanding. When students use manipulatives, they improve their imaging abilities by interacting with tangible 3D models, allowing them to mentally construct and manipulate geometric things more accurately.

Studies suggest that spatial reasoning skills can be enhanced through guided instruction and practice with concrete and virtual manipulatives (Hwang & Lin, 2021). These skills are particularly important in disciplines that require geometric modeling, such as engineering and computer graphics. Hence, improving students' spatial abilities through manipulatives can lead to better performance in geometry and related fields. The research discusses four core theories that illuminate how kids learn geometry and other mathematical concepts: Visualization Theory, Social Constructivist Theory, Zone of Proximal Development (ZPD), and APOS Theory.

Visualization Theory in Geometry Learning

Mathematical visualization has long been used to explain abstract topics via symbols and diagrams. Since Bishop developed the concept in the 1970s, study has expanded to look into individual preferences for mathematical visualization and how they relate to spatial skills. According to Abrahamson et al. (2023), mathematical concepts, ideas, and methods contain numerous visual linkages that can be intuitively conveyed in a variety of ways. This emphasizes the importance of how these theories and approaches are applied, presented, and used to solve mathematical issues. Despite a long history of rejection or avoidance, math educators grew interested in the practical issues of mathematical visualization in the 1980s. According to Nielsen et al. (2020), the ability to perceive differently is a taught and constructed skill rather than an inherent process. Lafay et al. (2019) define visualization as a multifaceted process involving altering mental pictures and representations, connecting previously unknown concepts to gradually build comprehension. Teachers who use visualization techniques can promote mathematical ideas and improve student learning in the classroom. Schematic visualization uses diagrams to depict relationships and mental objects, which helps students grasp mental processes (Chatima, 2021).

Social Constructivist Theory and Zone of Proximal Development

According to constructivist ideas, pupils develop their own understanding of the world by reflecting on their experiences. Cognitive constructivists consider learning as the process of discovering meaning through individual interpretation, whereas social constructivists see learning as a socially mediated process that

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results in observable involvement. McRobbie and Tobin (1997) argue that social constructivism is sophisticated, with roots in humanism, educational tradition, and cognitive brain science. The socio-constructivist approach includes cognitive activity, cultural knowledge, tools and signals, and aided learning. Cognitive activity entails generating meaning from psychological occurrences that cannot be understood as characteristics of an individual unrelated to their surroundings. The employment of cultural tools and indicators has evolved cognitive activity, which influences and governs behavior through the mediation of environment and behavior. Vygotsky introduced the concept of the Zone of Proximal Development (ZPD), which is the gap between what a learner can do independently and what they can achieve with guidance from a more knowledgeable person. Van Compernolle (2021) divided cognitive activity into three domains: motivation and orientation, goal-oriented action, and operations. Pattern and orientation serve to arrange the subject within the world of objects. When students work in groups, differences of opinion due to individual ZPDs might lead to a common understanding. The use of manipulatives in instructional activities helps pupils reach their ZPD. Visualization is an important aspect of learning solid geometry concepts because it aids the cognitive process by which learners comprehend mathematical concepts.

APOS Theory in Geometry Learning

Dubinsky (1948) established APOS Theory, which describes how actions are integrated into processes before being enclosed as mental objects inside complicated cognitive frameworks (Oldenburg, 2024). Based on Piaget's idea of reflective abstraction, this theory proposes that individuals create their own thinking tools and logic, resulting in logical and mathematical structures. According to Powell and Kalina (2009), reflective abstraction supports the building of cognitive structures throughout the thought process. Piaget felt that knowing an item required both the subject and the body to act on it, implying an unbreakable bond between the two. This paradigm distinguishes between internalized operations and material actions, allowing for the identification of mental structures inside an activity and demonstrating how internalization and encapsulation lead to the APOS hypothesis. According to Marougkas et al. (2023), APOS Theory holds that a certain mathematical notion can make sense to individuals if they have the necessary mental framework. Abstract mathematical notions can be categorized as objects (structural) or operational processes. In APOS, action and process are operational concepts, while objects and schema are structural words. The theory emphasizes assisting students in utilizing their present mental models while also allowing for the formation of new models for managing advanced mathematics. The abbreviation APOS stands for Action, Process, Object, and Schema.

a. *Action*: A concrete or mental operation that may be repeated and that alters objects.
b. *Process*: A cognitive action that occurs wholly in the learner's mind and internalizes repeated acts into thought processes.
c. *Object*: The outcome of an operation; when a process is recognized as having control over which procedures and activities can be carried out, it eventually produces an object.

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d. *Schema*: A collection of actions, processes, objects, and other schemas linked by common principles that constitute a support system within an individual's mind to be used in problem-solving scenarios.

The goal of this study is to assess the efficacy of manipulatives in increasing Basic 8 students' 3D geometry achievement. The study focuses on how manipulatives affect students' performance, gender differences, perceptions, and knowledge of 3D geometry topics. The manipulatives utilized in this study include physical replicas of 3D shapes (cubes, pyramids, cylinders, and prisms), which were chosen for their relevance to the curriculum. These materials were utilized in guided classroom activities that required students to interact with the shapes, investigate their qualities, and apply their knowledge to problem-solving problems. The organized usage of these manipulatives was intended to improve students' spatial reasoning skills and conceptual understanding of geometry.

Research Hypothesis

The researcher proposed the following hypothesis to be tested at the confidence level of 95% (α=0.05)

H_1: manipulatives have no effect on students' 3D geometry achievement.

H_2: traditional method of teaching had no effect on students' 3D geometry achievement.

H_3: there is no significant difference between the performance scores of male and female students taught with manipulatives.

RESEARCH METHOD

Research Design

This study is experimental, with the researcher employing an explanatory sequential design, resulting in a mixed method research methodology in which quantitative data is collected before qualitative data. The qualitative data and analysis clarify and explain the statistical results by delving further into participants' opinions and attitudes, comprehension, and motivation (Abu & Toyon, 2021). The quantitative element of the inquiry was conducted using true-experimental procedures. This study consisted of two groups of students: the control group and the experimental group. The control group was taught using standard methods, whereas the experimental group was taught with manipulatives. Achievement tests (pre-test and post-test) were administered to both groups before and after the intervention treatment.

Population

The population of the study involved basic 8 students of Tanoso District of 2023/2024 academic year constituting 665 students. This students were taught plane shapes and properties in basic 7 as foundations to learning other geometry like solid shapes (Ministry of Education, 2020).

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Sample size and Sampling Technique

Power analysis was used to establish the optimum sample size for the study. Chang et al. (2021), used a meta-analysis of similar educational interventions to estimate a two-tailed confidence level ($\alpha = 0.01$) and an effect value of $d = 0.8$. The following parameters were used for the calculation: two-tailed t-test, effect size: $d = 0.8$, allocation ratio: N2/N1 = 1. This resulted in a needed sample size of 154 individuals from a total of 665 Basic 8 students. To ensure a representative sample, a multistage sampling procedure was used: Purposive Sampling: Seven schools were chosen from the district due to similar curriculum structures and accessibility. 22 pupils were picked from each of the designated schools to ensure an even distribution of participants. The final 154 students were randomly assigned to either the experimental (77 students) or control (77 students) groups. This ensured that each school was adequately represented in the overall study sample. The 22 participants were chosen using simple random sampling within each school, ensuring an unbiased sample. Students were grouped based on the random numbers generated using Excel. The first 154 students on the sorted list were chosen as participants and divided into groups. This stratified technique ensured a balanced and representative sample while preserving the experimental design.

Data Collection Instruments

Achievement Test

This study's achievement test was a modified version of the Manipulatives in Teaching Elementary Mathematics and Transformation Geometry and Orthogonal Views of Geometric Figures tests created by Kadwa and Alshenqeeti (2020) and Kerim (2014), respectively. The test was designed to assess students' 3D geometry achievement and was divided into two sections: an objective portion with 10 multiple-choice questions and three structured theory questions that required students to use problem-solving and analytical reasoning skills in geometry. It assessed their ability to calculate volume, surface area, and grasp 3D geometric relationships. Subject-matter experts assessed the test to confirm its validity and reliability before administering it to both control and experimental groups for pre-test and post-test assessments.

Questionnaire

A 12-item questionnaire was distributed to students to measure their perceptions and attitudes toward using manipulatives to learn 3D geometry. The questionnaire was based on earlier research by Murat Kahveci (2010), Shadaan and Eu (2013), and Yasar (2015). It was designed utilizing a five-point Likert scale, with students rating their agreement with items ranging from strongly disagree (1) to strongly agree (5). The questionnaire addressed the following aspects: Perceptions of manipulatives in learning geometry, and attitudes toward manipulatives To guarantee validity and reliability, the questionnaire was content-validated and piloted prior to final administration.

Structured Interview

Structured interview on students' level of understanding on the use of manipulatives as instructional tools for teaching and learning solid geometry was

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designed for the participants in the experimental group. This consists of six questions. These questions were modification items used in a semi-structured interview by Yılmaz Zengin (2017); Zengin and Tatar (2015) and Ponte et al. (2023). The questions reflected six the levels or qualities of Bloom's taxonomy of learning which included: knowledge, comprehension, application, analysis, synthesis and evaluation.

Measurement Indicators

The following measuring indicators were used to assess students' spatial reasoning and visualization skills in this study:

a. ***Mental Rotation Tasks***: Evaluating the ability to mentally manipulate 3D shapes and recognize their rotated versions.
b. ***Spatial Perception Tests***: Measuring the ability to differentiate between different spatial orientations and relationships.
c. ***Visualization Assessments***: Testing students' capacity to interpret and construct mental images of geometric shapes.
d. ***Surface Area and Volume Application***: Assessing students' ability to apply spatial reasoning skills in solving real-world geometry problems.

These indicators are consistent with earlier research on spatial thinking and visualization in mathematics education (Petersen and McNeill, 2013). Incorporating these measurements provides a thorough study of how manipulatives effect students' cognitive growth in geometry.

Validity and Reliability of instruments.

The solid geometry achievement test items were a modification of a Transformation Geometry and Orthogonal Views of Geometric Figures by Kadwa and Alshenqeeti (2020), and (Kerim, 2014). Yasar (2015), changed the attitude questionnaire design and used it to gather information about participants' attitudes toward the use of manipulatives. Items on the perception questionnaire were also altered from research conducted by Shadaan and Eu (2013), and Murat Kahveci (2010). These changes guaranteed the degree to which the items assessed the Constructs. The reliability of the achievement test was evaluated using the piloted data. The split semi-reliability strategy, which applies the Spearman-Brown process, was used in this study due to the nature of the test (Chen, 2022; Wang et al., 2024). This was done to establish the extent to which the test gadget gives a dependable result. The result for each question was recorded in SPSS v. 23 and a correlation analysis (Product Moment Correlation) for the pre-test and the post-test was obtained as $r = 0.683$ and $r = 0.801$ respectively.

The reliability coefficients for the pre- and post-tests were thus calculated to be 0.760 and 0.850, respectively, which according to Demir and Kaya (2022), must be greater than 0.60 to be employed in a study. The reliability tests of the questionnaire on students' attitudes and perceptions yielded Cronbach's alpha values of $\alpha = 0.786$ for attitudes and $\alpha = 0.807$ for perceptions.

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Data Analysis Procedure

Data from achievement tests, questionnaires, and structured interviews were analyzed using descriptive and inferential statistical techniques in SPSS v. 23. The analysis was carried out in three steps to comprehensively address the study's hypotheses and research objectives. The accomplishment test data were evaluated to determine the effect of manipulatives on students' 3D geometry achievement, as well as to compare male and female performance. The attitude and perception surveys were analyzed using descriptive statistics to compute means, standard deviations, and skewness in order to uncover trends in students' attitudes and perceptions of studying 3D geometry with manipulatives. Furthermore, in the structured interview, thematic analysis was used to organize participants' replies into themes such as conceptual comprehension, motivation, and learning problems.

Hypotheses Tested:

a. **H_1: *Manipulatives have no significant effect on students' 3D geometry achievement*.**
Paired Samples t-test was conducted using descriptive and inferential statistics to compare the pretest and post-test means scores of the experimental group (at $\alpha = 0.05$). The results of the test indicated $t = -12.115, \text{df} = 76$ and $\text{p}(0.000) < \alpha(0.05)$.
***Conclusion*:** This implies that the use of manipulatives in teaching 3D Geometry significantly improved students' 3D geometry achievements, hence we rejected the null hypothesis stated "Manipulatives have no effect on students' 3D geometry achievement".

b. **H_2: *The traditional method of teaching has no significant effect on students' 3D geometry achievement*.**
Paired Samples t-test was conducted using descriptive and inferential statistics to compare the pretest and post-test means scores of the control group (at $\alpha = 0.05$). The results of the test indicated $t = 0.383, \text{df} = 76$ and $\text{p}(0.703) > \alpha(0.05)$.
***Conclusion*:** This implies that the use of traditional method in teaching 3D Geometry did not significantly improve students' 3D geometry achievements, hence we failed to reject the null hypothesis stated "The traditional method of teaching has no significant effect on students' 3D geometry achievement"

c. **H3: *There is no significant difference between the performance scores of male and female students taught with manipulatives*.**
The Independent Samples t-test was conducted using descriptive and inferential statistics to compare to compare the pre-test and post-test means scores of the posttest results of male and female students in the experimental group (at $\alpha = 0.05$). The results of the test revealed $t = 0.483, \text{df} = 75$ and $\text{p}(0.630) > \alpha(0.05)$.
***Conclusion*:** This implied that is no gender-based performance difference between male and female students taught with manipulatives, we failed to reject the null hypothesis stated "There is no significant difference between the performance scores of male and female students taught with manipulatives."

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Table 1: Summary of Statistical Tests and Hypothesis Evaluation

Hypothesis	Test Used	Conclusion Criteria
H_1: Manipulatives have no significant effect on students' 3D geometry achievement.	Paired Samples t-test.	**H_1** was rejected since $p(0.000) < \alpha(0.05)$
H_2: Traditional method of teaching has no significant effect on students' 3D geometry achievement.	Paired Samples t-test.	**H_2** was accepted since $p(0.703) > \alpha(0.05)$
H_3: No significant significance difference between male and female students' means scores.	Independent Samples t-test.	**H_3** was accepted since $p(0.630) > \alpha(0.05)$

RESULTS AND DISCUSSION

Pretest Results the Two Groups

Table 2 presents the descriptive and inferential statistics of the pretest results for both control and experimental group. An independent samples t-test was used to determine the statistical differences between the means of the pre-test score of both groups at $\alpha = 0.05$. It is evident from Table 2 that there is a difference in the mean scores of the experimental and control group by 1.117 (thus, $15.501 - 14.384 = 1.117$). However, there is no statistical difference between the pre-test results of the experimental and control groups by their values ($\mathrm{t} = 0.538, \mathrm{df} = 75, \mathrm{p} = 0.592; p > \alpha$). This indicates that both groups had equivalent levels of 3D geometry knowledge before the implementation of the manipulatives.

Table 2: Independent Samples T-Test for Both Groups

Groups	N	Mean	St. Deviation	St. Error Mean	t-test	df	Sig.
Control Group	77	15.501	3.044	0.498	0.538	75	0.592
Experimental Group	77	14.384	2.978	0.486			

Paired Sample T-Test of The Experimental Group

Table 3 presents the descriptive and inferential statistics for the pretest and posttest scores of the experimental group. The results indicate pretest score for the experimental class with a mean of 14.338 and standard deviation of 2.963. And for the posttest, a mean score of 20.221 and standard deviation of 2.887. The posttest results yield higher mean score than the result of the pretest results. The difference in these means is 5.883 (thus, $20.221 - 14.338 = 5.883$). The results further revealed that, there was a statistically significant difference between the mean scores of the pretest and posttest of the experimental group. This is indicated by $t = -12.115, \mathrm{df} = 76$ and $\mathrm{p}(0.000) < \alpha(0.05)$. This implies that the use of manipulatives in teaching 3D Geometry improved students' 3D geometry achievements.

Table 3: Paired Sample T-Test of Experimental Group

Groups	N	Mean	St. Deviation	St. Error Mean	t-test	df	Sig.
Pretest	77	14.338	2.963	2.963	-12.115	76	0.000
Posttest	77	20.221	2.887	2.887			

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Paired sample t-test of the Control Group

Table 4 shows the descriptive and inferential statistics for the pretest and posttest performance of students who were taught using only the traditional method. The study reveals pretest score with a mean of 14.533 and standard deviation of 3.01131. And a mean of 14.338 and standard deviation of 2.96297 for the posttest score. The difference in the two means scores is 0.195 (thus, $14.533 - 14.338 = 0.195$), and this difference in favour of the pretest mean. However, the difference in the mean scores was not statistically significant. This is indicated by $t = 0.383, \mathrm{df} = 76$ and $\mathrm{p}(0.703) > \alpha(0.05)$. This implies that the use of traditional method in teaching 3D Geometry did not improve students' 3D geometry achievements.

Table 4: Paired Sample T-Test of The Control Group

Groups	N	Mean	St. Deviation	St. Error Mean	t-test	df	Sig.
Pretest	77	14.533	3.011	0.343	0.383	76	0.703
Posttest	77	14.334	2.963	0.338			

Independent Sample T-Test of Posttest Results of Males and Females of The Experimental Group

Table 5 the shows descriptive and inferential statistics for the posttest results of male and female students of the experimental group (participants taught with manipulatives). The results revealed a male posttest mean score of 20.356 and standard deviation of 2.963, and a female posttest mean score of 20.031 and standard deviation of 2.811. It is evident that the male posttest mean score was higher than the female posttest mean score with a difference of 0.325. however, this difference was not statistically significant. This is indicated by $p(0.630) > \alpha(0.05)$.

Table 5: Independent Sample T-Test of Posttest Results of Males and Females of The Experimental Group

	Gender	N	Mean	St. Deviation	St. Error Mean	t-test	df	Sig.
Post Test	Male	45	20.356	2.963	0.442	0.483	75	0.630
	Female	32	20.031	2.811	0.497			

Students' perceptions on learning 3D geometry with manipulatives

The responses of students from the questionnaire items on perception were examined using descriptive statistics, which included the mean, standard deviation, and skewness of the replies to the items. Table 6 shows the results of students' responses to the perception items about the usage of manipulatives in learning 3D geometry. Table 6 findings reveal the lowest mean response value of 3.62 and a standard deviation of 1.00, which represents the response to the statement "I felt more confident using the manipulatives during the lesson". This indicates that the students' level of agree and strongly agree was skewed toward agreement. The response to the statement "I was excited having the opportunity to learn 3D shapes using their original objects" had the highest response mean value of 4.81 with a standard deviation of 0.488. The overall mean of 4.40 suggests that participants generally agree with the assertions regarding their perceptions of the usage of

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manipulatives in learning 3D geometry. In general, the results from table 6 suggest that the experimental group had positive perceptions of the use of manipulatives when studying 3D geometry (solid shapes).

Table 6: Students' perceptions on learning 3D geometry using manipulatives

S/N	Item	Min.	Max.	Mean	Std. D	Skewness	
						Statistic	Std. Error
P1	I was excited having the opportunity to learn 3D shapes using their original objects	2	5	4.81	0.488	-3.245	.274
P2	The manipulatives used during the lesson helped improve my 3D geometry understanding.	3	5	4.75	0.463	-1.599	.274
P3	The manipulatives used during the learning process helped me remember concepts better	2	5	4.73	0.737	-2.757	.274
P4	The manipulatives made it easier to visualize properties of shapes.	2	5	4.77	0.510	-2.758	.274
P5	The physical objects make learning more fun and engaging	2	5	3.95	0.972	-0.600	.274
P6	The use of manipulatives has increased my interest in learning geometry.	1	5	4.21	0.991	-1.182	.274
P7	I felt more confident using the manipulatives during the lesson	2	5	3.62	1.000	-0.390	.274
P8	I was able establish the relationships between 2D and 3D geometry shapes.	3	5	4.34	0.528	0.141	.274
	Overall Mean			**4.40**			

Students' Attitudes towards learning 3D geometry with manipulatives

Table 7 presents the responses of students from the questionnaire items on attitudes which were examined using descriptive statistics, which included the mean, standard deviation, and skewness of the replies to the items. The findings of the descriptive analysis of students' attitudes regarding the use of manipulatives in 3D geometry instruction. The findings indicate that the mean value of the minimum reaction was 3.65, with a standard deviation of 1.190. The answers to the statement, "I enjoyed using manipulatives in learning 3D geometry," are shown here. This indicates that there was a bias towards agreement among the responders. The statement "I believe that using manipulatives is a valuable addition to teaching and learning 3D geometry" elicited the highest mean answer of 4.48 with a standard deviation of 0.620. The overall mean of 4.20 shows that opinions regarding the usage of manipulatives in 3D geometry instruction are broadly shared by participants. Generally, the results table 7 demonstrates that the experimental

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group's participants had good attitudes regarding using manipulatives to study 3D geometry (solid shapes).

Table 7: Students' attitudes towards learning 3D geometry using manipulatives

S/N	Item	Min.	Max.	Mean	Std. D	Skewness	
						Statistic	Std. Error
A1	I believe that using manipulatives is a valuable addition to teaching and learning 3D geometry.	3	5	4.48	0.620	-0.774	.274
A2	Using manipulatives made it easier for me to apply 3D geometry concepts to real-world situations.	2	5	4.13	0.732	-0.415	.274
A3	The teacher made learning geometry enjoyable during the teaching and learning process.	3	5	4.12	0.743	-0.192	.274
A4	I enjoyed using manipulatives in learning 3D geometry.	2	5	3.65	1.190	-0.342	.274
A5	Manipulatives made abstract geometry concepts more practical and understandable.	3	5	4.40	0.613	-0.498	.274
A6	The use of manipulatives has improved my spatial reasoning skills.	3	5	4.34	0.620	-0.372	.274
A7	I prefer to learning 3D geometry with manipulatives over traditional methods.	3	5	4.37	0.626	-0.452	.274
A8	I would recommend using manipulatives in teaching 3D geometry to other students.	2	5	4.07	0.951	-0.698	.274
	Overall Mean			4.20			

Students' Levels of Understanding and Motivation on 3D Geometry by the Use Manipulatives as an Instructional Approach.

Structured interviews for members of the experimental group were undertaken to investigate and ascertain the extent of students' comprehension and enthusiasm regarding 3D geometry through manipulatives. All the participants were given codes in the form *UM01, UM02, UM03, ... UM77*. The questions in understanding reflect the six-levels of understanding of the Bloom's Taxonomy and the Profile Dimension stated in the GES JHS Mathematics Syllabus. These levels include Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation.

Structured Interview Responses

All participants in the experimental group indicated that they were excited haven learnt the 3D geometry using their original object in the classroom. Some of their responses to the series of questions presented to them are stated below.

First Question

Can you name any three 3D shapes you know?

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All participants answered in the affirmative were able to named three 3D shapes. The respondents *UM01, UM06, UM9, UM10, UM11, UM22, UM77* named the same shapes but not all in the same order. They named the following:

Cylinder, Cone, Cube

UM02, UMO8, UM12, UM20, UM21, UM30, UM33, UM35, UM41, also named the following shapes:

Pyramid, Sphere, Prism

These responses from the participants show that all the participants from the experimental group have grasped the basic concepts of the 3D shapes when the manipulatives were used as the intervention strategy.

Second Question

How would you explain the difference between any two of the three 3D shapes you have named?

All respondents were also to explain the difference between any two of the shapes they named. 49 respondents named the same 3D shapes and some respondents who named *Cylinder, Cone and Cube* stated that:

The cylinder has two circular ends but the cone has only one circular end. The cone one side is a pointed like a pencil tip. (UM02)
The cylinder looks like milk tin, but the cube looks like a box. (UM08)
The cylinder has some round face but the cube has flat faces allover. (UM12)

These responses from the participants show that the use of manipulatives has increased the participants' level of visualization and spatial reasoning skills.

Third Question

Can you mention two for each, everyday objects that are shaped like the 3D shapes you named?

All the participants responded in the affirmative to this question. 54 respondents mentioned the same examples regardless of the order. Some mentioned the following:

Cylinder: milk tin, milo tin; Cone: funnel, traffic cone; Cuboid: trunk, chop-box. (UM01), (UM07), (UM11), (UM12), (UM13)

Sphere: football, globe; Cone: ice-cream cone, Christmas tree; Cube: ice-cube, water tank. (UM10), (UM15), (UM32), (UM34), (UM60), (UM61).

Students after the have the experience of manipulatives of 3D shapes can easily recognize the various 3D shape structures in the real-life phenomena.

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Fourth Question

How do you visualize and work with 3D shapes in your mind?

I visualize 3D shapes by looking at their base shape, the top parts and the sides. When I look at a 3D shape this way, then I will remember it name and properties. (UM75).

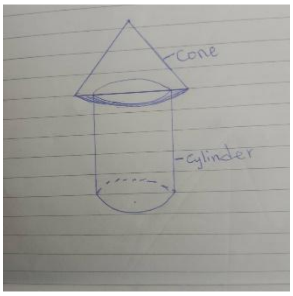
I have memorized the names of the 3D shapes we learned and their properties and some their formulas. so, when I see any 3D shape, I remember how to solve its question. (UM51).

Fifth Question

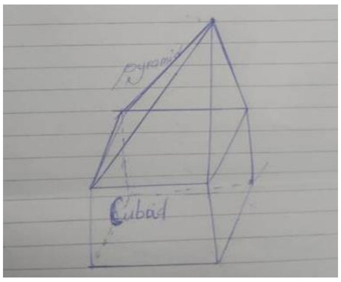
*Can you create a complex 3D shape using a combination of two different 3D shapes of those three shapes you have named**?***

Three of these respondents stated the following:

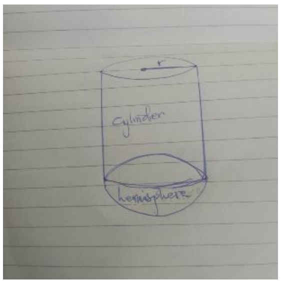
If I put a cone on top of the cylinder it will look like a hut: (UM16)



When I place a pyramid on a cuboid it becomes like a normal room: (UM17)



I can combine a cylinder and hemisphere to look like this: (UM45)



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This means that the participants demonstrated synthesis skill following the implementation of the manipulatives.

Sixth Question

How do you classify shapes that have similar properties but are fundamentally different?

In response to this question, 49 participants were able to express themselves with clear example. Some two participants' responses are as follows:(UM30), (UM32)

When I see that two shapes are a similar, I will think about their nets. This way I can tell if they all fall under prism or pyramid. (UM30).

I know that the prisms are have rectangular faces whiles the pyramids have triangular faces, cone is like a pyramid with a circular base and curved face. So, I know that if I want to calculate the volume of any of those 3D, I will find the area of the base and multiply it by the perpendicular height. (UM32).

Thes indicate that students' ability to distinguish between similar-looking objects has increased. Manipulatives improve memory retention, deepen conceptual knowledge, facilitate practical application, encourage analytical thinking, foster evaluative abilities, and stimulate creativity by offering concrete, hands-on experiences at each level of the taxonomy.

DISCUSSION

The results of the study show that manipulatives significantly enhanced students' achievement in 3D geometry. Before the intervention, both the control and experimental groups performed similarly on the pretest. However, following the intervention, the experimental group had significantly higher post-test scores than the control group, indicating that manipulatives improve students' comprehension and memory of 3D geometric ideas. This improvement is consistent with the findings of Hwang et al. (2016) and Lafay et al. (2019), who argue that interactive tools improve spatial thinking and problem-solving abilities. The use of manipulatives allowed students to visualize and manipulate 3D objects, resulting in an improved understanding of geometric properties. Students participated in hands-on activities that explored the links between 2D and 3D shapes, surface area, and volume. The enhanced spatial and analytical skills demonstrated in the experimental group highlight the value of manipulatives as a pedagogical tool in geometry education. The fact that there was no statistically significant difference in post-test performance between male and female students in the experimental group suggests that the effectiveness of manipulatives in teaching 3D geometry is gender-neutral, reinforcing the notion that both male and female students benefit equally from interactive learning strategies.

The study also looked at students' perceptions and attitudes about using manipulatives to learn 3D geometry. The questionnaire replies were extremely favorable, with students expressing excitement about using physical models to learn

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geometric ideas. Most students thought that manipulatives made studying more entertaining and helped them understand complex ideas. This is consistent with the findings of Goracke (2019), and Ponte et al. (2023), who argue that active involvement with manipulatives promotes a good learning experience and increases student motivation. Furthermore, students in the experimental group showed more confidence in problem-solving activities, implying that manipulatives not only improve understanding but also contribute to higher mathematical self-efficacy.

The structured interview results revealed that students who utilized manipulatives improved significantly across many Bloom's Taxonomy cognitive levels, including knowledge, understanding, application, analysis, synthesis, and evaluation. By physically handling 3D objects, students improved their spatial thinking and analytical skills. This is consistent with Pham (2015) and Palme (2019), who argue that manipulatives bridge the gap between abstract and concrete mathematical cognition. Furthermore, pupils who used manipulatives demonstrated improved problem-solving abilities and were able to apply geometric principles in real-world settings. This demonstrates the practical benefits of including manipulatives in the maths curriculum.

CONCLUSION

The findings of this study show that using manipulatives significantly improves students' achievement in 3D geometry at the elementary school level. The study's major goal was to determine the impact of manipulatives on students' three-dimensional geometry achievement, and the findings show that pupils in the experimental group outperformed those taught using standard methods. The improvement in students' spatial reasoning, imagery, and conceptual comprehension of 3D forms demonstrates how manipulatives can stimulate deeper cognitive engagement and improve geometric concept memory.

Pupils' imagery and spatial reasoning abilities were measured using indicators such as their ability to recognize and manipulate 3D geometric shapes, their performance in spatial visualization tasks, and their mental rotation and transformation of geometric objects. These indicators were assessed through pre-test and post-test evaluations, structured interviews, and questionnaire data.

Also, in assessing gender performance differences, the study found no statistically significant variation in the achievement scores of male and female students exposed to manipulatives. Both genders demonstrated similar improvements in their understanding of 3D geometry, suggesting that manipulatives are effective instructional tools that benefit all students regardless of gender. This finding supports the notion that active learning strategies, such as the use of manipulatives, can create inclusive learning environments where both male and female students develop strong spatial reasoning skills without bias.

Based on these findings, this study offers the following recommendations

Teachers should systematically integrate manipulatives into 3D geometry lesson plans to improve student engagement and comprehension. And also, schools should also provide adequate learning materials to facilitate the effective implementation of these instructional tools.

Educators should adopt gender-responsive teaching practices to guarantee that manipulatives assist both male and female pupils equally. This may include

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developing activities that cater to different learning styles, creating an inclusive classroom climate, and encouraging equal participation from all students. Further research should look into whether certain manipulative-based instructional strategies are more effective in improving spatial reasoning skills in male or female students.

Training workshops should be organized to provide mathematics teachers with the necessary abilities to properly integrate manipulatives into their teaching approaches. And also, educational leaders should consider modifying the mathematics curriculum to enhance hands-on learning experiences using manipulatives. This method will encourage active learning and develop students' spatial reasoning skills, which are required for higher-level mathematics and STEM-related areas.

The study also recommended further that future research should look into the long-term effects of manipulatives on students' recall of 3D geometry ideas. Researchers should explore the influence of manipulatives on students with various learning styles and cognitive abilities in order to understand how to best incorporate instructional methods to meet distinct learner requirements.

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