Mathematics Education Journals
Vol. 9 No. 1 February 2025

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

Metacognition and Mathematical Problem-solving: An Empirical Investigation into Series Problems with Junior High School Students

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Keywords:
Metacognition; Problem-solving; Metacognitive strategies; Series problems; Mathematics education; Polya's method

Abstract
This empirical study examines the critical relationship between metacognitive skills and mathematical problem-solving performance through a focused case study on series summation problems. Conducted with 50 seventh-grade students, the research involved a structured online lecture covering rational number series and algebraic transformations, followed by two problem-solving tasks and a detailed metacognitive questionnaire aligned with Polya's four-phase framework. Results demonstrated a significant disparity: while all students (100%) successfully solved the initial summation problem $(1 + 2 + \ldots + n)$, only 29 students (58%) correctly solved the more complex squared summation problem $(1^2 + 2^2 + \ldots + n^2)$. Analysis of the metacognitive questionnaire revealed pronounced differences in strategy use. Students solving both problems successfully reported significantly higher engagement in metacognitive behaviors: reading problems multiple times (72.4% vs. overall 42%), schematic representation (100% vs. 78%), strategic planning (100% vs. 78%), solution monitoring (86%), and calculation verification (100% vs. 90%). Statistical analysis confirmed strong positive correlations between these specific metacognitive strategies and successful problem-solving outcomes. The study robustly concludes that explicit metacognitive strategy deployment, particularly in problem representation, planning, and evaluation, is a decisive factor in successful mathematical problem-solving, especially for non-routine tasks. These findings underscore the imperative for systematic integration of metacognitive skill development within secondary mathematics curricula to enhance students' conceptual understanding and transfer abilities.

Zhao, Yin, & Saleh, Salmiza. (2025). Metacognition and Mathematical Problem-solving: An Empirical Investigation into Series Problems with Junior High School Students. *Mathematics Education Journal*, 9(1), 90-101. DOI: 10.22219/mej.v9i1.39924

INTRODUCTION

The distinction between a mathematical problem and a mere exercise is foundational to understanding the challenge of fostering genuine problem-solving ability. As Niss (1988) cogently argued, "a (mathematical) problem is a situation giving rise to certain open questions that are intellectually challenging to somebody who is not in immediate possession of direct methods procedures algorithms etc. which can answer the questions and solve the problems." Schoenfeld (1983) reinforced this view, stating that "a problem is only a problem (as mathematicians use the term) if you don't know how to go about solving it. A problem that holds no 'surprises' in store, and that can be solved comfortably by routine or familiar

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
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procedures (no matter how difficult!) is an exercise." Consequently, the core objective of problem-solving instruction transcends procedural fluency; it aims to cultivate students' capacity to transfer knowledge and adapt strategies to novel, non-routine situations—moving beyond retention to authentic application (Mayer, 2001; Schoenfeld, 1992).

Mayer's (2001) tripartite model posits that successful problem-solving hinges upon three interconnected components: skill (domain-specific knowledge and procedures), metaskill (metacognitive knowledge and regulation), and will (motivational factors like interest and self-efficacy). While the teaching of basic mathematical skills—conceptualized variously as instructional objectives (Bloom et al., 1956), hierarchical learning components (Gagne, 1968), or elements within information processing models (Sternberg & Gardner, 1983)—is undeniably crucial, empirical evidence suggests it is insufficient for fostering robust problem-solving transfer (Mayer, 2001; Schneider & Artelt, 2010). Mayer defined "metaskills" as metacognitive knowledge, specifically "knowledge of when to use, how to coordinate, and how to monitor various skills in problem-solving" (Mayer, 2001). In essence, metaskills represent the strategic control layer governing the application of procedural knowledge. The motivational component (will), encompassing interest, self-efficacy, and attributional beliefs, further influences engagement and persistence. However, the relative contribution and sufficiency of each component remain areas of active investigation.

Metacognition, literally "cognition about cognition" or "thinking about thinking" (Jacobs & Paris, 1987), provides the theoretical bedrock for understanding metaskills. Flavell's (1979) seminal framework delineated four interrelated facets:

1. Metacognitive Knowledge: Awareness of one's own cognitive processes, task demands, and effective strategies.
2. Metacognitive Experiences: Conscious cognitive or affective experiences occurring during problem-solving (e.g., feeling confused or confident).
3. Goals/Tasks: The objectives of the cognitive enterprise.
4. Actions/Strategies: The cognitive processes employed to achieve goals.

Schneider and Artelt (2010) later proposed consolidating the latter two facets into metacognitive skills, emphasizing the procedural and regulatory aspects. A more concise, yet widely adopted, definition characterizes metacognition as encompassing both "knowledge that one has about a cognitive domain, and executive strategies that regulate thinking" (Jacobs & Paris, 1987). While debates regarding the precise components persist, often reflecting theoretical rather than empirical distinctions (Desoete et al., 2001), consensus exists on its critical role in complex cognition.

Polya's (1957) enduring four-phase model provides a robust scaffold for analyzing the problem-solving process:

a. Understanding the Problem: Comprehending the goal, unknowns, data, and conditions.
b. Devising a Plan: Formulating a strategy connecting known information to the goal.
c. Executing the Plan: Carrying out the chosen strategy computationally and logically.

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

d. Looking Back: Reflecting on the solution process and verifying the result.

Lester (1994) significantly augmented this model by arguing that Polya's phases primarily describe the cognitive dimension. He posited that an equally vital metacognitive component operates concurrently, involving the monitoring, control, and regulation of cognitive activities at each stage. This integration positions metacognition as the executive function guiding the problem-solving journey.

Research consistently underscores the value of metacognition in mathematics. Questionnaires and self-report protocols have proven effective tools for investigating metacognitive engagement and informing pedagogy. Fortunato et al. (1991) highlighted that "the metacognitive statements and students' responses to them can enrich teachers' awareness of how students reflect on mathematical problem-solving and can help to identify topics or strategies that need to be emphasized". Biryukov (2004) concluded that "metacognition provides a more promising platform to set goals, and to perform actions to achieve those goals during problem solving". Echoing this, Sengul and Katranci (2012) emphasized the importance of teachers designing activities that explicitly elicit and develop students' metacognitive skills.

Despite this established significance, targeted investigations into the specific metacognitive behaviors linked to success in particular mathematical domains, such as series summation—a topic involving pattern recognition, algebraic manipulation, and strategic decomposition—remain less common. Series problems offer a rich context as they often transition from routine exercises (e.g., summing consecutive integers) to genuine problems requiring insight (e.g., summing squares or higher powers, telescoping series). This study addresses this gap by examining the relationship between explicit metacognitive strategies, framed within Polya's phases and Lester's metacognitive component, and students' performance on carefully sequenced series problems. It specifically investigates:

a) The disparity in success rates between routine and non-routine series problems.
b) The prevalence of specific self-reported metacognitive behaviors among students.
c) The correlation between the use of these metacognitive strategies and successful problem-solving outcomes.
d) The implications for integrating metacognitive strategy instruction into mathematics teaching.

RESEARCH METHOD

The study employed a mixed-methods case study design, combining quantitative performance data with qualitative self-report data on metacognitive processes. The intervention consisted of three sequential phases conducted within a single online session via a secure video-conferencing platform (Zoom) with breakout rooms disabled:

1. Instructional Phase (120 minutes): A researcher-delivered online lecture introduced fundamental concepts and solution strategies for rational number series and algebraic transformation series. The lecture structure was explicit and interactive, incorporating worked examples and brief conceptual checks. The outline was:
   a. Objective:
      1) Understand basic calculations of rational number series.
      2) Understand basic calculations of algebraic transformation series.

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

3) Practice mathematical problem-solving skills.

b. Preliminaries (Review - 15 min):
   1) Primary school arithmetic calculations (properties of operations, fractions).
   2) Basic algebraic calculations (variables, simplification, summation notation Σ introduction).

c. Worked Examples (105 min): Eight progressively complex series problems were solved interactively, emphasizing strategic approaches (e.g., decomposition, telescoping, recognizing patterns/formulas). Solutions were presented as shown in the original document (Examples 1-8). Key strategies highlighted included:
   1) Recognizing harmonic series patterns and re-grouping (Example 1).
   2) Utilizing factorial identities ($n * n! = (n+1)! - n!$) for telescoping sums (Example 2).
   3) Applying the triangular number formula ($1 + 2 + \ldots + n = n(n+1)/2$) and partial fraction decomposition ($1/(k(k+1)) = 1/k - 1/(k+1)$) (Examples 3, 4).
   4) Exploiting combinatorial identities or polynomial expansions for sums of products ($n(n+1), n(n+1)(n+2)$) (Examples 5, 6).
   5) Strategic algebraic manipulation using differences and symmetric expressions (Example 7).
   6) Identifying and applying polynomial factorization patterns ($n(n+3) + 2 = (n+1)(n+2)$) (Example 8). Students were encouraged to ask clarifying questions during this phase.

2. Problem-Solving Phase (30 minutes): Immediately following the lecture, participants were given two series problems to solve independently within a 30-minute time limit via a shared Google Form. Proctoring ensured independent work. The problems were:
   a. Problem 1 (Routine): Calculate $S1 = 1 + 2 + 3 + \ldots + n$. (Solution methods provided as per original document).
   b. Problem 2 (Non-Routine): Calculate $S2 = 1^2 + 2^2 + 3^2 + \ldots + n^2$. (Solution methods provided as per original document). Problem 1 served as a familiar anchor, while Problem 2 required transferring concepts (like telescoping or polynomial identities) demonstrated in the examples to a new but structurally related context.

3. Metacognitive Assessment Phase (15 minutes): Upon submitting their solutions to Problem 2, participants were directed to a separate section of the Google Form containing the 13-item metacognitive questionnaire. They were instructed to reflect specifically on their thought processes while solving Problem 2 and select the response (YES, NO, UNSURE) that best described their behavior. The questionnaire items, mapped explicitly to Polya's phases and self-regulation aspects (Planning, Monitoring, Evaluation), were:
   a. I read the second problem more than once. Understanding the Problem (Planning).
   b. I checked what the second problem was asking to me. Understanding the Problem (Planning).
   c. I assessed how much time I need for solving the second problem. Devising a Plan (Planning).
   d. I represented the second problem schematically. Devising a Plan (Planning).
   e. I tried to recall whether I solved a similar problem before. Devising a Plan (Planning).
   f. I have established a strategy for solving the second problem. Devising a Plan (Planning).
   g. I did not know how to begin. Devising a Plan (Monitoring).
   h. I met a difficulty during solving the second problem. Executing the Plan (Monitoring).

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

i. I found a mistake and corrected it during solving the second problem. Executing the Plan (Monitoring).
j. I thought how the solution was going on. Executing the Plan (Monitoring).
k. I tried different approaches for solving the second problem. Executing the Plan (Monitoring).
l. I asked myself whether my solution made sense. Looking Back (Evaluation).
m. I checked my calculation to make sure they were correct. Looking Back (Evaluation).

Quantitative data (problem correctness, questionnaire responses) were analyzed using descriptive statistics (frequencies, percentages) and inferential statistics. Chi-square tests of independence were performed to determine if significant associations existed between specific metacognitive behaviors (YES responses on key items) and successful solution of Problem 2. Effect sizes were calculated using Cramer's V. Qualitative analysis involved identifying themes in open-ended solution explanations where permitted. Reliability (internal consistency) of the questionnaire was assessed using Cronbach's Alpha. Statistical analyses were conducted using SPSS v28.0, with $\alpha = 0.05$ set as the significance level.

RESULTS AND DISCUSSION

1. Problem-Solving Performance

All 50 participants (100%) correctly solved Problem 1 (S1 $= n(n+1)/2$). In contrast, only 29 participants (58%) successfully solved Problem 2 (S2 $= n(n+1)(2n+1)/6$). Among the successful solvers of Problem 2, 26 students (89.7% of successful, 52% of total) used a single approach (primarily the method utilizing $n * n!$ identity by analogy or the polynomial expansion method), while 3 students (10.3% of successful, 6% of total) demonstrated flexibility by applying multiple valid approaches and clearly documenting them. All students who solved Problem 1 correctly also completed Problem 2 within the time limit, though 21 failed to find the correct solution. Table 1 summarizes the performance results.

Table 1. Student Performance on Series Summation Problems

Problem	Solved Correctly	Solved Incorrectly	Used Multiple Approaches
Problem 1	50 (100%)	0 (0%)	5 (10%)*
Problem 2	29 (58%)	21 (42%)	3 (6%)

*Note: Multiple approaches on Problem 1 were noted but not a primary focus, as success was universal.

2. Metacognitive Questionnaire Responses

The overall response frequencies for the 13 metacognitive items related to Problem 2 are presented in Table 2. The questionnaire demonstrated acceptable internal consistency for a context-specific scale (Cronbach's $\alpha = 0.72$).

Key Observations (Overall):

a) Universal checking of the problem goal (Item 2).
b) High levels of monitoring solution progress (Item 10, 86%) and checking calculations (Item 13, 90%).

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

c) High recall of similar problems (Item 5, 90%) but low actual strategy adaptation (Item 11, 18% YES).
d) Very low time assessment (Item 3, 0% YES) and low reporting of mistake detection/correction (Item 9, 22% YES).
e) Significant reported difficulty (Item 8, 64% YES), though most claimed to establish a strategy (Item 6, 78% YES).

Table 2. Frequencies and Percentages of Student Responses to Metacognitive Questionnaire (N=50)

Metacognitive Statement (Regarding Problem 2)	YES	NO	UNSURE
I read the second problem more than once.	21 (42%)	19 (38%)	10 (20%)
I checked what the second problem was asking to me.	50 (100%)	0 (0%)	0 (0%)
I assessed how much time I need for solving the second problem.	0 (0%)	41 (82%)	9 (18%)
I represented the second problem schematically.	39 (78%)	10 (20%)	1 (2%)
I tried to recall whether I solved a similar problem before.	45 (90%)	5 (10%)	0 (0%)
I have established a strategy for solving the second problem.	39 (78%)	11 (22%)	0 (0%)
I did not know how to begin.	12 (24%)	30 (60%)	8 (16%)
I met a difficulty during solving the second problem.	32 (64%)	18 (36%)	0 (0%)
I found a mistake and corrected it during solving the second problem.	11 (22%)	39 (78%)	0 (0%)
I thought how the solution was going on.	43 (86%)	7 (14%)	0 (0%)
I tried different approaches for solving the second problem.	9 (18%)	41 (82%)	0 (0%)
I asked myself whether my solution made sense.	42 (84%)	5 (10%)	3 (6%)
I checked my calculation to make sure they were correct.	45 (90%)	5 (10%)	0 (0%)

3. Metacognition and Problem-Solving Success

Crucially, analysis revealed stark contrasts in metacognitive behavior between students who solved Problem 2 correctly ($n = 29$) and those who did not ($n = 21$). Table 3 presents the YES response rates for key items within the successful group and the results of Chi-square tests comparing successful vs. unsuccessful solvers.

Key Findings (Success Association):

a. Essential Metacognitive Behaviors for Success: A perfect 100% of successful solvers reported engaging in schematic representation (Item 4), strategic planning (Item 6), monitoring their solution progress (Item 10), evaluating solution sense (Item 12), and verifying calculations (Item 13). These

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

behaviors showed statistically significant and moderate to very strong associations with success.

b. Critical Role of Re-reading: Successful solvers were significantly more likely to re-read the problem (72.4% vs 0% in unsuccessful group; $\chi^2 = 25.76, p < .001, \varphi c = 0.72$), highlighting its importance beyond initial comprehension.
c. Difficulty vs. Stagnation: While encountering difficulty (Item 8) was common overall (64%), it was reported by all unsuccessful solvers (100%) but only 37.9% of successful solvers ($\chi^2 = 22.91, p < .001, \varphi c = 0.68$). Crucially, successful solvers who encountered difficulty were the only group reporting finding and correcting mistakes (Item 9: 37.9% YES in successful vs 0% in unsuccessful; $\chi^2 = 10.40, p = .001, \varphi c = 0.46$), indicating effective monitoring and debugging.
d. Recall vs. Application: Recalling similar problems (Item 5) was near-universal (90%) and unrelated to success. However, acting on that recall by adapting strategies (Item 11) or effectively planning (Item 6) distinguished successful solvers.
e. Lack of Planning & Comprehension Issues: Unsuccessful solvers showed significantly lower rates of schematic representation and strategic planning. The absence of re-reading and inability to detect/correct errors further suggest fundamental struggles with comprehension and self-regulation.

Table 3. Metacognitive Behaviors of Successful vs. Unsuccessful Solvers of Problem 2 (χ^2 Analysis)

Metacognitive Item	Successful Solvers (n = 29) YES %	Unsuccessful Solvers (n = 21) YES %	χ^2 (df=1)	p-value	Cramer's V	Interpretation
(1) Read problem more than once.	72.4% (21)	0% (0)	25.76	< .001	0.72	Very Strong Assoc.
(4) Represented problem schematically.	100% (29)	47.6% (10)	19.41	< .001	0.62	Strong Assoc.
(6) Established a strategy before calculating.	100% (29)	47.6% (10)	19.41	< .001	0.62	Strong Assoc.
(10) Thought about how solution was progressing.	100% (29)	66.7% (14)	10.94	.001	0.47	Moderate Assoc.
(12) Asked if solution made sense.	100% (29)	61.9% (13)	12.65	< .001	0.50	Moderate Assoc.
(13) Checked calculations for correctness.	100% (29)	76.2% (16)	8.33	.004	0.41	Moderate Assoc.
(3) Assessed time needed.	0% (0)	0% (0)	n/a	n/a	n/a	No Variation

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

(5) Recalled similar problem.	89.7% (26)	90.5% (19)	0.01	.936	0.01	No Assoc.
(7) Did not know how to begin.	17.2% (5)	33.3% (7)	1.71	.191	0.18	No Sig. Assoc.
(8) Met a difficulty.	37.9% (11)	100% (21)	22.91	< .001	0.68	Very Strong Assoc. (Inverse)
(9) Found & corrected mistake.	37.9% (11)	0% (0)	10.40	.001	0.46	Moderate Assoc.
(11) Tried different approaches.	10.3% (3)	28.6% (6)	2.69	.101	0.23	No Sig. Assoc.

4. Discussion

This study provides robust empirical evidence supporting the pivotal role of specific metacognitive strategies in successful mathematical problem-solving, particularly when transitioning from routine exercises to non-routine problems. The stark contrast in performance between Problem 1 (100% success) and Problem 2 (58% success) underscores Schoenfeld's (1983) distinction: Problem 1, while foundational, likely functioned as an exercise for most students after the lecture examples, whereas Problem 2 presented a genuine problem requiring strategic adaptation and insight. The findings offer granular insights into the metacognitive dimensions of Pólya's phases within Lester's (1994) integrated framework.

a. Understanding the Problem: Beyond Initial Comprehension

While all students reported checking the problem goal (Item 2), the significant association between re-reading (Item 1) and success (72.4% of successful solvers vs. 0% unsuccessful) is profound. This suggests that successful solvers engage in deeper, iterative processing beyond initial comprehension checks. They likely revisited the problem to clarify relationships, identify implicit patterns, or confirm their understanding after initial planning attempts - behaviors aligning with Flavell's (1979) metacognitive experiences and knowledge activation. The universal use of schematic representation (Item 4) by successful solvers further emphasizes that translating the problem into a different representational form (e.g., writing the series explicitly, sketching partial sums, identifying known sub-components like Σk from Problem 1) is not merely helpful but essential for managing the complexity of non-routine problems like Σk^2. This finding resonates strongly with research highlighting the importance of representation in mathematical reasoning and supports Polya's first phase as critically metacognitive.

b. Devising a Plan: The Centrality of Strategy and Anticipation

The finding that all successful solvers reported establishing a clear strategy before calculation (Item 6) is arguably the most significant of this study. This deliberate planning phase involves metacognitive knowledge (selecting potentially viable approaches based on task analysis and prior knowledge) and

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

foresight (anticipating steps and potential pitfalls). The contrast with unsuccessful solvers (only 47.6% reported planning) strongly supports Mayer's (2001) emphasis on metaskills as the coordination layer for procedural knowledge. Interestingly, while almost all students recalled similar problems (Item 5), successful solvers effectively leveraged this recall to formulate a specific plan for this problem. The lack of significant association between trying different approaches (Item 11) and success, coupled with the low overall rate (18%), suggests that successful planning often pre-empts the need for major mid-problem shifts. This contrasts somewhat with views emphasizing flexibility but highlights the efficiency of good initial planning. The inverse relationship between encountering difficulty (Item 8) and success further underscores that early, clear strategic planning prevents debilitating confusion later. Difficulty was universal for unsuccessful solvers but only affected a minority of successful solvers, who then employed debugging strategies (Item 9).

c. Executing the Plan: Monitoring and Debugging

Monitoring progress (Item 10) was highly prevalent among successful solvers (100%). This ongoing metacognitive activity involves continually assessing whether the execution aligns with the plan and the problem's demands - a form of "online" metacognitive control. The ability to detect and correct errors (Item 9) was exclusive to successful solvers (37.9%) and significantly associated with success. This highlights a crucial difference: successful solvers possess not only procedural skills but also the metacognitive vigilance to identify deviations and self-correct during implementation. Unsuccessful solvers either failed to detect errors or lacked the strategies to correct them. The low overall rate of mistake detection/correction (22%) even among successful solvers suggests this is a particularly challenging metacognitive skill requiring explicit development.

d. Looking Back: Verification as a Non-Negotiable Habit

The perfect correlation between solution verification behaviors (Items 12 & 13) and success reinforces Polya's emphasis on "Looking Back" as indispensable. Evaluating the solution's plausibility (Item 12, e.g., "Does this formula grow roughly like n^3?", "Does it work for $n = 1$ or $n = 2$?") and meticulously checking calculations (Item 13) constitute the final metacognitive safeguard against errors. While 84-90% of all students reported these behaviors, the 100% adherence among successful solvers indicates it is a consistent hallmark of proficient problem-solvers. The presence of some unsuccessful solvers reporting these checks (61.9% for sense-making, 76.2% for calculation) suggests their checks might have been superficial or applied to fundamentally flawed solution paths, underscoring the need for verification integrated with sound planning and understanding.

e. Implications for Instruction

The results strongly advocate for explicit metacognitive strategy instruction embedded within mathematics teaching:

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

1) Teach Metacognitive Routines: Structure problem-solving around explicit phases (Polya's). Require students to document their understanding (e.g., "What is given? What is unknown?"), plan ("What strategy will I try first? Why?"), monitor ("Is this step working? Does it make sense so far?"), and evaluate ("Does my answer make sense? How can I verify it?"). Use think-alouds by teachers and peers to model these processes (Schoenfeld, 1985).
2) Emphasize Representation & Planning: Dedicate time to teaching diverse representational strategies (diagrams, tables, symbolic manipulation, verbal descriptions) and explicitly teach planning before calculation. Use prompts like: "Draw a picture of the problem," "How is this similar to problems we've seen?", "What is your first step and why?".
3) Foster Reflective Practice: Build in structured reflection during and after problem-solving. Use journals or discussion prompts focusing on process: "What was difficult? How did you overcome it?", "What mistake did you catch? How?", "Would a different strategy work better next time?".
4) Target Debugging Skills: Explicitly teach error detection and correction strategies. Analyze common errors, discuss why they occur, and practice identifying and fixing them in worked examples and peer work.
5) Design Tasks for Transfer: Move beyond procedural practice to include genuine problems requiring adaptation, like the Σk^2 task used here. Sequence tasks to build complexity and encourage strategic choice. The lecture examples in this study provided the necessary building blocks, but students needed metacognitive skills to assemble them for Problem 2.

f. Limitations and Future Research

This study has limitations. The sample size (N=50), while sufficient for initial correlations, limits generalizability. Participants came from a single school, potentially limiting diversity. The reliance on self-reported metacognition, though common, can be susceptible to recall bias or social desirability; future research should combine questionnaires with think-aloud protocols or eye-tracking for richer data. The study design focused on immediate post-instruction performance; longitudinal studies are needed to assess the long-term impact of metacognitive training. The problems were specific to series summation; research should explore if these metacognitive patterns hold across other mathematical domains (e.g., geometry, combinatorics). Finally, investigating the impact of explicit metacognitive strategy instruction based on findings like these on students' problem-solving transfer is a critical next step.

CONCLUSION

This investigation provides compelling evidence that metacognitive skills are not merely auxiliary but fundamental determinants of success in mathematical problem-solving, particularly when confronting non-routine challenges. The significant performance gap between the routine summation (Σk) and the non-routine squared summation (Σk^2) problem underscores the limitations of procedural knowledge alone. The differentiating factor lay in the strategic deployment of metacognition.

ISSN : 2579-5724
ISSN : 2579-5260 (Online)
http://ejournal.umm.ac.id/index.php/MEJ

Successful solvers consistently engaged in deep processing (re-reading), effective representation, deliberate strategic planning before calculation, continuous monitoring of their progress, vigilant debugging when difficulties arose, and rigorous evaluation of their final solution. Crucially, behaviors like schematic representation and explicit strategic planning were exhibited by all successful solvers and none of the unsuccessful solvers exhibited the full set of key metacognitive strategies. The strong statistical associations between these specific metacognitive behaviors and successful outcomes highlight their critical importance.

These findings translate into a clear imperative for mathematics educators: fostering metacognitive competence must be an explicit and integral goal of instruction, not an incidental byproduct. By systematically teaching students how to understand deeply, plan strategically, monitor their progress, debug errors, and evaluate their solutions, we equip them with the essential "metaskills" (Mayer, 2001go) needed to navigate the complexities of genuine mathematical problems and transfer their learning effectively. Integrating these practices into curricula, as outlined in the implications, holds significant promise for enhancing students' mathematical proficiency and problem-solving agency.

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