

## Hypothetical Learning Trajectory Using Straws to Teach Subtraction to Elementary School Students

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| <b>Corresponding author:</b>   | <b>Abstract</b>  |
| Titah Pinasthi<br><a href="mailto:titahpinasthi04@gmail.com">titahpinasthi04@gmail.com</a>                                   | This study developed a context-based Hypothetical Learning Trajectory (HLT) using straws as concrete manipulatives to support elementary students' understanding of two-digit addition and subtraction. Employing a Design Research approach, the study consisted of an initial design, two cycles of teaching experiments, and a retrospective analysis. The HLT was constructed through sequential activities that guided students from concrete representations to symbolic reasoning related to place value, addition with and without carrying, and subtraction with and without borrowing. The findings indicate that straw-based manipulatives effectively supported students in developing the concept of regrouping, enhancing calculation accuracy, and strengthening conceptual understanding, particularly during carrying and borrowing processes. Across Activities A–D, the actual learning trajectory aligned with the hypothetical pathway, with notable benefits for students with lower initial abilities. Overall, the results confirm that bundling-based concrete media such as straws offer a meaningful and feasible instructional alternative for early-grade mathematics learning. |
| <b>Keywords:</b><br>Hypothetical Learning Trajectory; subtraction operations; straws; elementary school mathematics learning |  |

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### Introduction

Basic arithmetic skills such as subtraction are important topics in mathematics education at the elementary level. Mastery of these concepts is essential for the development of advanced mathematical competencies, such as multiplication, division, and an understanding of basic algebra and geometry. Many studies confirm that conceptual errors that occur in the early stages, if not addressed immediately, will have a long-term impact on students' mathematics achievement (Cai et al., 2021; Mulligan et al., 2020; Siegler & Lortie-Forgues, 2020). The *National Council of Teachers of Mathematics* (NCTM) emphasizes that early-age students need to be given the opportunity to understand the meaning of number operations through concrete and contextual activities in order to form a strong conceptual foundation (NCTM, 2020). In line with this, PISA 2022 data shows that most Indonesian students still have difficulty solving simple numerical problems involving two-digit addition and subtraction, indicating a weak mastery of basic arithmetic (OECD, 2023).

Teaching two-digit addition and subtraction, both with and without *carrying* and *borrowing*, is a critical stage in basic mathematics learning because it requires an understanding of *place value*, procedural accuracy, and problem-solving strategies. (Carvalho, 2022) shows that students who do not understand the conceptual difference between operations without *carrying* and with *carrying* tend to make repeated procedural errors. This difficulty is particularly apparent in two-digit subtraction with *regrouping*, which is often caused by a weak understanding of the concept of *place value*, rather than simply an inability to follow procedural steps. Therefore,

pedagogical strategies are needed that can bridge the conceptual transition in a gradual and meaningful way, utilizing concrete contexts and manipulative media that have been proven effective in helping students understand number relationships (Rittle-Johnson & Schneider, 2020).

One critical aspect in teaching addition and subtraction operations is the ability to distinguish between addition with and without *carrying*, and subtraction with and without *borrowing*. A conceptual understanding of the *regrouping* process (*carrying* and *borrowing*) is essential to ensure the accuracy of students' calculation strategies (Van de Walle et al., 2022). However, early elementary school students are often taught only formal algorithms without understanding the meaning of the process (Fuson & Murata, 2021). This condition causes students' knowledge to be superficial and fragmented, so that they are unable to adjust their strategies when faced with variations in questions or different numerical contexts. As a result, students tend to rely on memorizing mechanical steps without understanding the mathematical reasoning behind each procedure. According to a study by (Kullberg et al., 2022), an approach that focuses only on procedures results in long-term misconceptions. This shows that the failure of the procedural approach lies in its inability to build meaningful relationships between *place value* concepts and number operations, while the conceptual approach encourages flexibility in thinking and a more lasting understanding of numbers. Therefore, there needs to be an approach that pays attention to children's thinking development and brings them closer to mathematical meaning.

In practice, the concept of *regrouping* is often presented abstractly, making it difficult for early grade students to understand. To overcome this problem, *Hypothetical Learning Trajectory* (HLT) has been widely used in various studies as a framework for designing meaningful learning that emphasizes the sequence of learning activities to enable students to construct their understanding gradually (Bakker, 2020; Gravemeijer & Terwel, 2021). On the other hand, straws as concrete tools have high potential to help students understand the processes of *regrouping* and *borrowing* explicitly because of their nature that can be easily grouped and moved physically. Concrete contexts such as straws or bundles of units and tens have been proven to improve students' comprehension of the meaning of *regrouping*, especially in two-digit operations (Widjaja et al., 2021).

However, previous studies tend to discuss HLT as a learning design framework or the use of concrete manipulatives as a representational strategy separately. The systematic integration of HLT and concrete manipulative contexts, particularly straws, in teaching two-digit subtraction with *borrowing* has not been explored in depth. In fact, through the design of an HLT-based learning path that deliberately utilizes the straw context, students not only perform manipulative activities but are also guided to reflect on the strategies and mathematical meaning behind the *regrouping* process. Thus, there is a research gap that emphasizes the need to develop and test concrete manipulative-based HLT for two-digit subtraction learning, as an effort to bridge the abstraction of concepts with meaningful and contextual learning experiences (Van Nes & Doorman, 2020).

To ensure the effectiveness of learning, a systematic and structured instructional design is required, one of which is through the *Hypothetical Learning Trajectory* (HLT) framework. In the context of this study, HLT is positioned as an analytical and instructional design tool that serves to map learning objectives, design a sequence of mathematical activities, and anticipate possible student responses and strategies during the learning process (Simon et al., 2022). Through HLT, teachers and researchers can understand students' thinking development more deeply and identify critical points where conceptual difficulties arise, especially in two-digit subtraction operations. As a learning development research framework, HLT plays an important role in directing the design of context-based manipulative activities to be in line with how students

construct mathematical understanding. This approach allows for a more systematic analysis of the interaction between activities, student strategies, and the learning objectives to be achieved. In line with previous findings, HLT designed with manipulative contexts in mind has been shown to be effective in improving basic mathematical concept understanding and supporting meaningful learning (Clements & Sarama, 2020; Dougherty et al., 2023; Van den Heuvel-Panhuizen et al., 2021).

Previous research shows that the development of *Hypothetical Learning Trajectory* (HLT) plays an important role in basic mathematics learning, particularly in addition and subtraction operations. Learning success is greatly influenced by the clarity of the HLT stages and their relevance to real-world contexts that are close to students' experiences (Fauzan et al., 2022). Research by (Mutaqin et al., 2023) suggests that HLT designed through gradual *place value* stages using concrete objects and worksheets can predict student responses and help teachers prepare anticipatory strategies. Furthermore, (Stephan & Akyuz, 2023) in ZDM Mathematics Education documented how HLT facilitates the shift of students' strategies from concrete to abstract through structured learning interventions. These findings are reinforced by (Simon et al., 2020), who emphasize that HLT design plays an important role in arranging a logical sequence of learning activities that are in line with children's cognitive development. Recent research by (Maulida et al., 2025) also shows that the application of HLT can improve elementary school students' basic mathematical problem-solving skills by emphasizing the interconnection between concepts.

Although various studies have examined the effectiveness of HLT and the use of concrete tools separately, studies that specifically integrate HLT into the learning of two-digit addition and subtraction operations with concrete manipulatives such as straws are still limited. Most HLT research has focused more on topics such as geometry, single numbers, or patterns, while the *regrouping* strategy in two-digit operations is rarely the main object of learning path development (Leinwand et al., 2022). On the other hand, empirical research on the effectiveness of concrete aids such as straws in facilitating understanding of *regrouping* has been conducted, but not many have been explicitly designed within the HLT framework (Putri et al., 2020). Therefore, this study contributes to filling this gap by developing and examining a concrete and contextual manipulative-based HLT to support the understanding of *regrouping* in two-digit addition and subtraction operations in elementary school.

This study aims to develop a *Hypothetical Learning Trajectory* (HLT) based on the context of straws as a manipulative medium in teaching two-digit addition and subtraction operations to elementary school students. The main objective of this article is to design and evaluate a learning trajectory that can facilitate students' understanding of the concept of *regrouping* through meaningful activities. The implication of this research is to provide an alternative to concrete context-based learning designs that can strengthen students' conceptual understanding, while also serving as a reference for teachers in designing more interactive, contextual, and appropriate mathematics learning strategies for the cognitive development characteristics of elementary school students.

## Research Methods

This study uses a *Design Research* approach that refers to the *Design Research* framework developed by (Gravemeijer P., 2006). This approach was chosen because it allows for the continuous development, testing, and refinement of learning designs in a real classroom context. Operationally, the research was carried out in three main stages, namely initial design, teaching experiments, and retrospective analysis. These three stages focused on designing and testing the *Hypothetical Learning Trajectory* (HLT) in learning addition and subtraction operations in elementary school by utilizing concrete media in the form of straws. Thus, *Design Research* not only functions as a methodological framework but also as a means to ensure that

the developed HLT is relevant, applicable, and responsive to how students construct their understanding of the concept of *regrouping* in learning practice.

The initial stage of the research, namely the initial design, focuses on the preparation of the HLT, which forms the basis for the direction of learning. At this stage, the researchers reviewed various literature on the common difficulties faced by students in understanding number operations and integrated the context of straws as a concrete representation. Straws were chosen because they are manipulative and easy for children to understand, especially in representing the concept of numbers as objects that can be physically added or subtracted. The results of the literature review and the identification of learning obstacles (Fauzi, 2020; Fuadiah D., 2017) were used as the basis for designing the sequence of learning activities, learning objectives, and students' HLT predictions, which were outlined in a learning portfolio in the form of Student Worksheets and observation sheets.

After the preliminary design stage, the research continued with the implementation of the teaching experiment stage, which was divided into two cycles. The first cycle aimed to test the feasibility of the initial HLT design and identify possible obstacles or gaps in its application. Data were collected through direct observation during the learning process and reflective discussions with teachers. Based on the findings in the first cycle, improvements were made to several aspects of the HLT and learning tools. In the second cycle, the revised design was reimplemented. The main focus was to observe how the straw context helped students understand addition and subtraction operations conceptually, for example, how students showed numbers by reducing the number of straws, or how they interpreted the addition of different numbers visually.

After implementation, a retrospective analysis was conducted to evaluate the effectiveness of the HLT that had been applied. At this stage, the collected data, such as portfolios in the form of worksheets, observation notes, visual documentation, and interviews with students and teachers, were analyzed qualitatively using the approach (Miles A. M. and Saldaña, J., 2014), which included data reduction, data presentation, and conclusion drawing. This analysis process aims to see the extent to which actual learning is in line with or differs from the designed HLT, as well as to understand the dynamics of learning that occur in the classroom. The researcher also identified important moments in the learning process that showed changes in students' understanding, for example, when they began to be able to explain the difference between adding two-digit numbers without *carrying* and with *carrying*, and subtracting two-digit numbers without *borrowing* and with *borrowing* through the context of straws.

This research was conducted at Wonoplintahan 2 Public Elementary School, a public elementary school located in a rural area in East Java, Indonesia. The subjects of this study were first-grade students with an average age of 7 years. Before the research was conducted, research approval was obtained from the principal and classroom teachers, and written consent was collected from parents. The students' identities were kept confidential to ensure ethical standards, and student participation was voluntary without academic consequences.

Data collection in this study was conducted through formative assessment and in-depth observation of the students' mathematics learning process. During the learning process, additional data was obtained from learning portfolios in the form of Student Worksheets and teacher observations of how students guessed when completing tasks in HLT. This process allowed researchers to dynamically align HLT with students' understanding development. HLT adjustments were made continuously to ensure that each activity could trigger the targeted conceptual understanding. The activities and questions designed based on HLT are presented in detail in Table 1.

**Table 1. Activities and task items for HLT**

| Activity   | Sub-activity  | Task Item   |
|--|---|---|
| A. Identifying the initial context of straws as concrete objects | A. 1 Grouping straws into bundles of 10 straws each (tens).   | Students tie every 10 straws into 1 bundle and count how many bundles of tens are formed.                             |
|  | A. 2 Interpreting straws that are less than 10 and not bundled as units.  | Students group straws that are less than 10 into the “unit” category and state the number of units in numerical form. |
| B. Identifying the <i>place value</i> of two-digit numbers       | A. Representing two-digit numbers with straws.  | Students are asked to arrange numbers such as 23 using bundles (tens) and individual straws.                          |
| C. Understanding the concept of adding two numbers               | C.1 Finding the sum of two-digit numbers without <i>carrying</i> when the sum of the ones is $< 10$ .                         | Students perform addition such as $21 + 32$ by combining two groups of straws.  |
|  | C.2 Finding the sum of two-digit numbers by <i>carrying over</i> when the unit result is $\geq 10$ .                          | Students try $28 + 17$ : the units become 15 (10 is made into a new bundle) and 5 remains as the units                |
| D. Understanding the concept of subtraction of two numbers       | D.1 Finding the subtraction of two-digit numbers without <i>borrowing</i> where the units result is $\geq 0$ .                | Students solve $54 - 23$ by directly taking the units and tens from the straw representation.                         |
|  | D.2 Finding the subtraction of two-digit numbers by <i>borrowing</i> when the units result is $< 0$ before <i>borrowing</i> . | Students try $42 - 19$ : not enough units $\rightarrow$ break down 1 bundle of tens into 10 units.                    |

Table 1 presents four main activities, namely Activities A, B, C, and D, which form a single *Hypothetical Learning Trajectory* (HLT) design. Activities A, C, and D each consist of several sub-activities that are designed sequentially to support the understanding of the concepts of two-digit addition and subtraction. Each activity functions as an interrelated conceptual tool, starting from reinforcing *place value* and number representation to applying arithmetic operations.

This sequence of activities is effective because it follows the development of students' thinking from concrete to abstract. Activity A prepares the foundation of *place value* as a cognitive prerequisite, so that students are not immediately confronted with symbolic operations. Activity B serves as a representational bridge that links concrete manipulation with number

symbols. Next, Activities C and D introduce addition and subtraction operations gradually, starting from cases without *borrowing* or *carrying* over to situations that require *regrouping*. This sequence allows students to build understanding progressively, while also providing space for teachers to provide different types of scaffolding according to student ability. HLT predictions for each activity are formulated by considering the differences in responses of high, medium, and low ability students, which are summarized in Table 2.

**Table 2. HLT assumption**

| Activity | Sub-activity | HLT assumption   |
|----------|--------------|--|
| A        | A.1          | Students hypothesize that every 10 straws can be tied into 1 bundle, so they can be counted as tens.   |
|          | A.2          | Students hypothesize that straws that do not reach 10 are called units and can be counted separately from bundles of tens.   |
| B        | B            | Students assume that numbers such as 23 consist of 2 bundles of straws (tens) and 3 single straws (units).   |
| C        | C.1          | Students assume that to add, units are added to units, tens are added to tens, without the need to make new bundles if the result of the units is less than 10.                    |
|          | C.2          | Students guess that if the sum of the units reaches 10 or more, then 10 units can be made into a new bundle and added to the tens.   |
| D        | D.1          | Students guess that subtraction can be done directly by taking a number of tens and units if the subtrahend is smaller than the number being subtracted from in the same position. |
|          | D.2          | Students hypothesize that a number such as 23 consists of 2 bundles of straws (tens) and 3 single straws (units).  |

The data collection techniques in this study included direct observation using structured observation sheets, video and photo documentation of learning activities, portfolios in the form of student work (worksheets), and interviews with students and teachers after learning. The observation procedure followed the approach used by (Fauzi, 2020; Zuliana N. A. and Aji, N. P., 2025), which emphasized observing students' interactions with concrete tools and the problem-solving strategies they used. Data analysis was strengthened by triangulation of sources and collaboration between researchers in the reflection process. With this approach, the study not only aimed to test the effectiveness of straw media in mathematics learning but also to develop a deeper understanding of how elementary school students construct number operation concepts.

Data analysis in this study was conducted qualitatively by applying the source triangulation technique, as described by (Patton, 2002). Source triangulation is a data analysis strategy that utilizes various sources of information to test the consistency of findings, enrich interpretation, and minimize researcher bias. In the context of this study, the data sources used included direct observation during the learning process, portfolios in the form of Student Worksheets that recorded student activities and work results, photo documentation that captured important moments in the activities, and interviews that provided an in-depth picture of the students' views and experiences.

Each data source was systematically analyzed and compared to identify similarities, differences, and relationships between the information obtained. This process allows researchers to confirm the validity of findings while capturing nuances that might be missed if only one type

of data were relied upon. Through the application of triangulation, the understanding of student responses, engagement, and development within the *Hypothetical Learning Trajectory* (HLT) framework becomes more complete, in-depth, and verified, thereby maintaining the quality of the research results.

## Result and Discussion

In this study, we designed a *Hypothetical Learning Trajectory* (HLT) on subtraction operations for first-grade elementary school students, consisting of four activities (Activities A-D). Each activity used a worksheet. In addition, students were also given straws and rubber bands. Each activity is described in the following section.

### A. Activity A: Identifying the initial context of straws as concrete objects

This activity serves as the foundation for understanding *place value* by using straws as a concrete representation. Students are asked to group straws into bundles of ten and identify the remaining straws as units. In the first cycle, some students were not consistent in forming bundles of ten straws, indicating that the concept of tens was still understood intuitively. After the teacher reemphasized and clarified the instructions in the second cycle, the majority of students began to be able to distinguish between tens and units more systematically.

The teacher showed a handful of straws and asked:

**Teacher:** “If we count these one by one, it will take a long time. How can you make it easier?”

**Students:** “Tie them in bundles of ten first, ma'am.”

**Teacher:** “Why ten?”

**Students:** “Because ten is one ten.”

Teacher-student interactions demonstrate an important process of meaning negotiation in the formation of *place value* concepts. When students state that ten straws form a ten, there is a shift from a strategy of counting one by one to a more efficient grouping strategy. This finding is in line with (Van Nes & Doorman, 2020), which confirms that manipulative experiences serve as a cognitive bridge in the transition from informal strategies to a more structured understanding of *place value*. However, unlike the findings of (Van Nes & Doorman, 2020), which reported consistency in grouping from the early stages, in this study, students still required teacher intervention in the first cycle to achieve conceptual stability. This difference indicates that the classroom context and students' initial experiences influence the speed of conceptual internalization.



**Figure 1.** Students grouping straws into bundles of ten and single straws in Activity A.

Figure 1 shows the process of students grouping straws into bundles of tens and single straws as an initial representation of *place value*. When compared to previous studies that used similar manipulatives, this activity shows a similar pattern of development, but with a more

explicit need for scaffolding in the early stages. When students begin to verbally say “this is a ten,” it marks the beginning of symbolic internalization, as predicted in the HLT learning trajectory, where concrete activities gradually lead to the formation of mental and symbolic representations.

### B. Activity B: Identifying the *place value* of two-digit numbers

In this activity, students are asked to compose two-digit numbers, such as 23, using bundles of tens and straws representing ones. This activity serves as a conceptual bridge between the manipulative experience in Activity A and the symbolic representation of numbers. In general, in both learning cycles, students were able to represent two-digit numbers, but a notable difference was seen in the level of consistency and ability to relate concrete representations to number symbols.

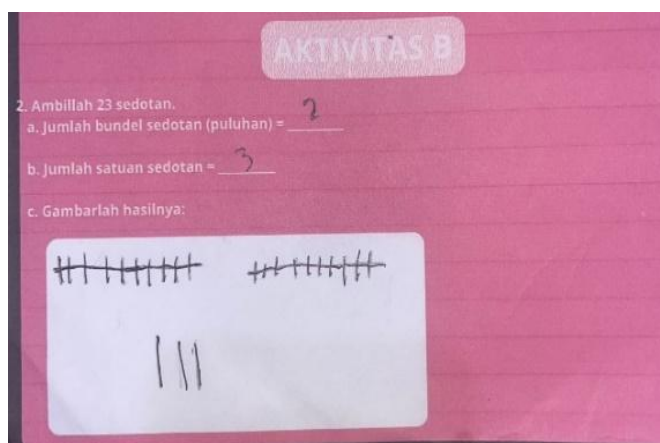


Figure 2. Work produced by high-achieving students

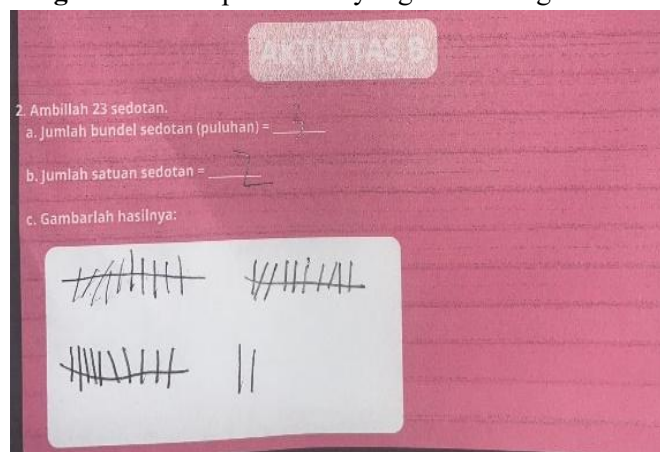


Figure 3. Work results of low-ability students

Figure 2 shows the work of high-ability students who are able to convert number symbols into bundles of tens and ones independently, as well as explain the mathematical reasoning behind the composition of these numbers. This indicates that students have developed a structural understanding of *place value*, where the first digit is understood as the number of tens and the second digit as the number of ones. In contrast, Figure 3 shows the work of low-ability students who still often swap digits, for example reading 23 as “thirty-two.” Cognitively, this error indicates that students still rely on the verbal sequence of memorized numbers rather than on *place value* structure. Digits are processed as a series of symbols without integrated quantitative meaning, so that the position of digits is not yet understood as *carrying* different values.

These findings reinforce previous research results which state that understanding of *place value* develops through repeated and reflective manipulative experiences, not through mere memorization of symbols (Mutaqin et al., 2023). The inability of low-ability students to maintain the tens-units sequence indicates that the transition process from concrete to symbolic representation is still incomplete. Thus, this activity emphasizes the importance of repeating manipulative activities accompanied by reflective discussion so that students can shift from verbal sequence-based thinking to a structural understanding of *place value* in line with the expected learning trajectory in HLT.

### C. Activity C: Understanding the concept of adding two numbers

This activity is divided into two sub-activities, namely two-digit addition without *carrying* and two-digit addition with *carrying (regrouping)*, which are designed to reveal the gradual development of students' understanding in accordance with the HLT pathway.

In Sub-Activity C.1, students learn two-digit addition without *carrying*, when the units result is less than 10. The majority of students were able to combine two groups of straws, then count the tens and ones directly without the need to form new bundles. The strategy revealed by the students, namely combining the ones first because it was considered easier, showed flexibility in calculation strategies and an initial understanding of the independence of the operation results from the grouping sequence. These findings are in line with (Mutaqin et al., 2023), which reported that in the early stages of HLT, students tend to develop efficient informal strategies when the manipulative context is still in line with the number structure encountered. However, differences remain apparent in low-ability students who still recalculate from zero, indicating that understanding of *place value* has not been fully internalized.

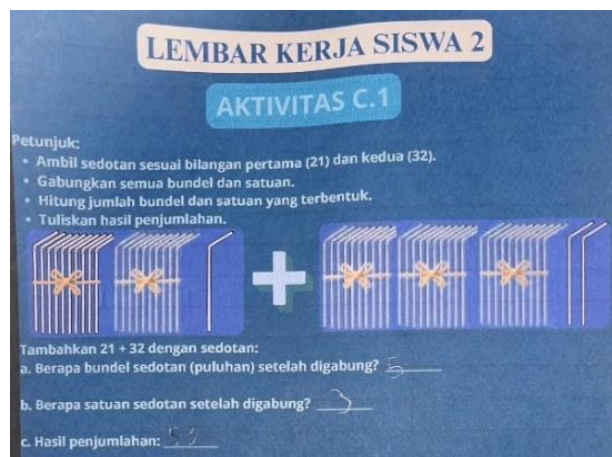
The teacher asked what they would do in this activity:

**Teacher:** “ $21 + 32$ . What do you do first?”

**Students:** “Combine the ones first, then the tens.”

**Teacher:** “Why not the tens first?”

**Students:** “The result is the same, but it's easier to do the ones first.”



**Figure 4.** Students' work on two-digit addition without *carrying* (Activity C.1)

Figure 4 shows that students can combine two groups of straws and count tens and ones without making new bundles.

Sub-activity C.2 aims to find the sum of two-digit numbers by keeping the ones digit  $\geq 10$ . This is a critical point in learning.

Students must understand that when the ones digit reaches  $\geq 10$ , a new bundle of 10 ones must be made.



**Figure 5.** The *regrouping* process in two-digit addition with *carrying* (Activity C.2)

Figure 5 shows how students form new bundles of ten units and integrate them into groups of tens. When compared to the findings (Carvalho, 2022), this pattern shows strong consistency: the bundling process is not merely a procedural step, but a means for students to understand the conceptual restructuring of numbers. High-ability students are able to regroup independently, while moderate-ability students need prompting to realize that “too many units” must be converted into tens. Meanwhile, the confusion of low-ability students indicates that without continuous concrete support, *regrouping* is easily perceived as a mechanical rule disconnected from the meaning of *place value*.

Overall, the findings in Activity C show that the designed HLT is coherent with students' actual learning trajectories, especially in the transition from addition without *carrying* to addition with *carrying*. Compared to the studies by (Carvalho, 2022) and (Mutaqin et al., 2023), the results of this study confirm that the integration of manipulative contexts in HLT not only helps students follow procedures but also strengthens their structural understanding of numbers. However, differences in the level of scaffolding needs among students emphasize the importance of adapting HLT to be more responsive to variations in the cognitive abilities of lower-grade students.

#### **D. Activity D: Understanding the concept of subtraction of two numbers**

Subtraction activities are designed in stages, starting from subtraction without *borrowing* to subtraction with *borrowing* as a learning path that increases in complexity. In Sub-Activity D.1, students are presented with two-digit subtraction without *borrowing*, where the unit result is still positive.

The teacher asks students about the tens or units bundle to be taken:

**Teacher:** “54 – 23. What do we need to take?”

**Student:** “Take 3 units first, then two bundles of tens.”

Teacher-student interactions show that most high- and medium-ability students are able to subtract directly using physical representations of straws by taking units first, then tens.

However, low-ability students still rely on the orderly arrangement of straws, indicating that *place value* structure has not been fully internalized and still requires strong visual support.

Sub-activity D.2 is the most challenging stage because it requires an understanding of the concept of *borrowing* when the number of units is insufficient. In cases such as  $42 - 19$ , students must understand that one bundle of tens can be broken down into ten units.

The teacher facilitates trigger questions:

**Teacher:** “We have 2 ones, but we have to subtract 9. What can we do?”

**Average ability students:** “Borrow tens... so break the bundle.”

**Low ability students:** “We can't, ma'am, there aren't enough tens...”

The teacher then demonstrates breaking the bundle, until the low ability students begin to realize the transformation “1 ten  $\rightarrow$  10 ones”.



Figure 6. The *borrowing* process in two-digit subtraction (Activity D.2)

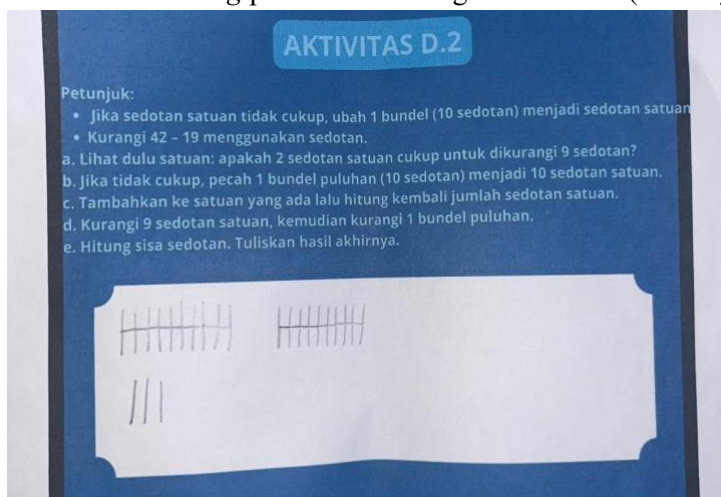


Figure 7. Student work

Figure 6 shows how students break down a bundle of tens into ten units to solve cases where there are not enough units. Figure 7 shows the results of the students' work. High-ability students can understand the idea of *borrowing* as a transformation of *place value*. Medium-ability students can understand after performing concrete manipulations. Low-ability students tend to fail to understand that tens can be converted into units.

These findings not only confirm previous research emphasizing the importance of concrete manipulatives in understanding *borrowing*, but also extend it by showing how explicitly designed HLT trajectories can map differences in scaffolding needs among students. Thus, this study expands on previous findings by showing that the systematic integration of HLT and straw manipulatives allows for the identification of critical points of conceptual failure in two-digit subtraction, particularly in *place value* transformation, which has not been explored in detail in previous studies.

Subtraction poses more complex conceptual challenges than addition. The research findings show that the concept of *borrowing* can be meaningfully understood when students observe and perform the process of changing a bundle of tens into ten units themselves. This reinforces the findings of a study (Van den Heuvel-Panhuizen et al., 2021) which confirms that informal strategies tend to be more stable when students are directly involved in manipulating concrete objects. Thus, the success of subtraction learning is not solely determined by the accuracy of the procedure, but by the conceptual experience students have during the *place value* transformation process.

Overall, Activities A–D show a strong correspondence between the designed HLT and the actual HLT in the classroom. The use of straws allows students to move gradually from concrete execution to symbolic representation, in line with the principles of Realistic Mathematics Education and the concept of emergent modeling. However, this study has several limitations that need to be considered. First, time constraints in learning meant that not all students, especially those with low abilities, had adequate opportunities for manipulative exploration. Second, the research subjects were limited to a specific classroom context and grade level, so the findings need to be generalized with caution. Third, the context of using straws as a concrete medium is highly dependent on the teacher's skills in providing scaffolding, so the effectiveness of HLT may differ when applied to different school contexts or student characteristics. Reflection on these limitations opens up opportunities for further research to test manipulative-based HLT in a broader context and with a longer learning duration.

## Conclusion

This study shows that the development of a context-based *Hypothetical Learning Trajectory* using straws can facilitate elementary school students' conceptual understanding of two-digit addition and subtraction operations. Through a series of systematic activities, students gradually move from concrete representations to a more mature symbolic understanding of *place value*, *carrying*, and *borrowing*. The use of straws proved effective in helping students construct number structures and visualize meaningful unit-to-tens transformations. The results of the teaching experiment in two cycles showed that the actual learning trajectory was in line with the designed trajectory, although low-ability students required more intensive scaffolding.

However, this study still has limitations in terms of subject scope, implementation duration, and the use of only one type of manipulative media. Therefore, further research is recommended to test similar HLTs in more diverse classroom contexts and using other manipulatives, such as Dienes blocks, number cards, or interactive digital media. Expanding the context and variety of manipulatives has the potential to enrich the HLT model and strengthen the generalization of findings on basic arithmetic learnings in elementary schools.

## Reference

- Bakker, A. (2020). *Design Research in education: A practical guide for early career researchers*. Routledge.
- Cai, J., Hwang, S., Jiang, C., & Silver, E. A. (2021). Mathematical thinking and reasoning: A review of theory and research. *Journal for Research in Mathematics Education*, 52(3), 221–256. <https://doi.org/10.5951/jresmetheduc-2021-0056>

- Carvalho, D. M. (2022). Understanding *regrouping* through concrete modeling. *Mathematics Education Research Journal*, 34(3), 457–471.
- Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). Routledge.
- Dougherty, B. J., Flores, A., & Jordan, C. C. (2023). Integrating hypothetical learning trajectories with lesson study: Enhancing teaching practices. *Mathematics Teacher Educator*, 11(2), 134–152. <https://doi.org/10.5951/mathteeduc.11.2.0134>
- Fauzan, A., Andita, C. D., Rada, G., Zafirah, A., & Abdullah, A. H. Bin. (2022). Developing RME-Based Learning Trajectory for Teaching Addition to A Dyscalculia Student in Elementary School. *Jurnal Didaktik Matematika*, 9(1), 39–58. <https://doi.org/10.24815/jdm.v9i1.25340>
- Fauzi, I. (2020). *Desain didaktis operasi hitung penjumlahan dan pengurangan pecahan di kelas 5 sekolah dasar*. Repository Universitas Pendidikan Indonesia. <https://repository.upi.edu/50067>
- Fuadiah D., N. F. and S. (2017). *Hypothetical Learning Trajectory* pada pembelajaran bilangan negatif berdasarkan teori situasi didaktis di sekolah menengah. *Mosharafa: Jurnal Pendidikan Matematika*, 6(1), 77–88. <https://journal.institutpendidikan.ac.id/index.php/mosharafa/article/view/425>
- Fuson, K. C., & Murata, A. (2021). Blending cultural models and mathematics teaching: Moving from algorithms to understanding. *ZDM Mathematics Education*, 53(1), 107–121. <https://doi.org/10.1007/s11858-020-01193-5>
- Gravemeijer P., K. and C. (2006). *Design Research from a learning design perspective BT - Educational Design Research* (pp. 17–51). Routledge.
- Gravemeijer, K., & Terwel, J. (2021). Realistic mathematics education as a new theory of mathematics education. *Educational Studies in Mathematics*, 107(1), 145–161. <https://doi.org/10.1007/s10649-021-10043-2>
- Kullberg, A., Runesson Kempe, U., & Marton, F. (2022). Teaching with variation: Research-based strategies for effective learning. *Scandinavian Journal of Educational Research*, 66(2), 203–220. <https://doi.org/10.1080/00313831.2021.1952432>
- Leinwand, S., Dougherty, B. J., & Chval, K. B. (2022). Mathematics instruction and tasks that matter. *Educational Leadership*, 79(4), 20–27. <https://www.ascd.org/el/articles/mathematics-instruction-and-tasks-that-matter>
- Maulida, D. W., Mahmudah, M. H., & Nuryadin, N. (2025). Problem-solving ability in realistic mathematics education based on *Hypothetical Learning Trajectory*. *JP2M (Jurnal Pendidikan Dan Pembelajaran Matematika)*.
- Miles A. M. and Saldaña, J., M. B. and H. (2014). *Qualitative Data Analysis: A Methods Sourcebook* (3rd ed.). SAGE Publications.
- Mulligan, J., English, L., & Watson, J. (2020). Whole-number knowledge and early arithmetic strategies in young learners. *Mathematics Education Research Journal*, 32(4), 531–550. <https://doi.org/10.1007/s13394-019-00277-1>
- Mutaqin, E. J., Herman, T., Wahyudin, W., & Muslihah, N. N. (2023). *Hypothetical Learning Trajectory* in *Place value* Concepts in Elementary School. *Mosharafa: Jurnal Pendidikan Matematika*, 12(1), 125–134. <https://doi.org/10.31980/mosharafa.v12i1.1313>
- NCTM. (2020). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.
- OECD. (2023). *PISA 2022 Results (Volume I): The State of Learning*. <https://www.oecd.org/pisa/publications/pisa-2022-results.htm>
- Patton, M. Q. (2002). *Qualitative Research and Evaluation Methods* (3rd ed.). Sage Publications.
- Putri, R. I. I., Zulkardi, & Hartono, Y. (2020). Designing learning trajectories of *place value* using bundle context. *Journal on Mathematics Education*, 11(2), 157–168. <https://doi.org/10.22342/jme.11.2.11753.157-168>

- Rittle-Johnson, B., & Schneider, M. (2020). Developing conceptual and procedural knowledge. *Educational Psychologist*, 55(1), 1–14.
- Siegler, R. S., & Lortie-Forgues, H. (2020). Conceptual knowledge of fractions and decimals. *Journal of Educational Psychology*, 112(3), 459–472. <https://doi.org/10.1037/edu0000374>
- Simon, M. A., Tzur, R., Heinz, K., & Kinzel, M. (2020). Hypothetical learning trajectories in mathematics education. *Mathematics Education Research Journal*, 32(1), 65–87.
- Simon, M. A., Tzur, R., Heinz, K., Kinzel, M., & Smith, M. (2022). Explicating a learning trajectory for linear measurement. *Journal for Research in Mathematics Education*, 53(2), 134–159. <https://doi.org/10.5951/jresematheduc-2022-0032>
- Stephan, M., & Akyuz, D. (2023). Design-based research and HLT development for arithmetic. *ZDM Mathematics Education*.
- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2022). *Elementary and Middle School Mathematics: Teaching Developmentally* (11th ed.). Pearson.
- Van den Heuvel-Panhuizen, M., Kolovou, A., & Robitzsch, A. (2021). Diagnosing students' informal strategies for subtraction. *Educational Studies in Mathematics*, 107(2), 279–300. <https://doi.org/10.1007/s10649-021-10044-1>
- Van Nes, F., & Doorman, M. (2020). Students' reasoning on *place value* using realistic contexts. *International Journal of STEM Education*, 7(1), 1–18. <https://doi.org/10.1186/s40594-020-00240-9>
- Widjaja, W., Groves, S., & Doig, B. (2021). Enhancing primary students' number sense through contextualized tasks. *Mathematics Education Research Journal*, 33(2), 299–316. <https://doi.org/10.1007/s13394-020-00324-3>
- Zuliana N. A. and Aji, N. P., E. and H. (2025). Pendekatan PMRI berbantuan House Counting untuk operasi hitung penjumlahan dan pengurangan pada bilangan cacah kelas II. *Inspiramatika*, 17(1), 33–45. <https://ejournal.unisda.ac.id/index.php/Inspiramatika/article/view/8644>