Cost Minimization Policy for Manufacturer in a Supply Chain Management System with Two Rates of Production under Inflationary Condition

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ABSTRACT

This article presents a production inventory model for deteriorating goods with two different rates of production. The manufacturer starts manufacturing the items at a lower rate to avoid a huge investment at the initial stage and reduce the products' holding cost. However, when the stock level reaches a prefixed level, he switches on to a higher production rate to avoid shortage caused by an insufficient stock of the items. Moreover, the impact of inflation and the time value of money on the manufacturing system's cost is considered here, which harms any business by reducing the value of an investment with time. We determined the optimum production times at both the low and high production rates by minimizing the system's total cost. Numerical examples illustrated the applicability of this proposed model. Sensitivity analysis studied the effect of changes in the parameters associated with this model on the optimal decision variables. This numerical experiment was done in LINGO 18.0 software. Results showed that the production strategy taken by the manufacturer helped reduce his total cost.

1. Introduction

Recently, the marketing or business environment is becoming highly competitive. Most businesses should adopt various strategic and tactical plans to sustain their existence. In this regard, a vital issue for any manufacturing firm is to fix an optimum production rate. A high manufacturing rate harms the business economy by investing huge capital for this process and incurring the holding and deterioration costs. On the other hand, an insufficient stock due to a low producing rate may result in a shortage, incurring penalty cost. Therefore, various authors studied this issue while developing their models. Datta [1] framed an inventory model considering the products' quality-dependent production rate, where the rate decreases when the quality improves. Saha and

Chakrabarti [2] developed an Economic Production Quantity model (EPQ), assuming that the production rate remains almost constant up to a certain period. After that, the rate decreases due to machinery fault, lethargy, or the personnel for continuous work. Alfares [3] assumed a finite production rate in their model. Another study [4] considered the demand-dependent production rate while developing an EPQ model. This study assumed the demand rate as a linearly increasing function of time; therefore, the producing rate is also a linear function. Roy and Samanta [5] and Bhowmick and Samanta [6] considered the two production rates in an inventory model with deteriorating items. Two studied [7], [8] discussed the constant production rate, which showed a decrease in the inventory system's total cost with the increase in this parameter. Different studies [9] also explained the constant production rate, which finds an increase in the inventory system's joint total profit with the increase in it. Keshavarzfard, et al. [10] framed a production- inventory model where the manufacturer's production rate varies with the power demand rate.

Besides the production rate, another critical factor affecting a company's financial system is inflation and money's time value. Money is losing the purchasing capacity of certain products day by day due to a high rate of inflation [11], [12]. Therefore, it forces one to spend more on buying a product and can reduce the value of an investment. An inventory model developed by Uthayakumar and Palanivel [13] projected the effect of inflation which showed a significant increase in the cycle length, the order quantity, and the total cost with an increase in the net discount rate of inflation. Khurana et al. [14] constructed a production inventory model for deteriorating products, which also showed its adverse effect on the system's total cost. Mondal et. al. [15] developed a production repairing inventory model under an inflationary environment. They formulated this model as an optimal control problem and found a negative impact of inflation on the system's profit. Some other studies [16], [17], [18] also find a rise in the production cost with the increase in the inflation rate.

The deterioration of any product is defined as the damage or the loss of utility partially or fully, resulting in the loss in the marginal value of this item. Most physical goods, such as agricultural products, foods, beverages, and pharmaceuticals goods deteriorate in their normal storage period. Several studies considering the deterioration of products showed that the inventory cost increases with the increase in the deterioration rate [19], [20], [21], [22], [23]. A study by Saha & Chakrabarti [24] postulated a constant rate of deterioration in their model. Prasad and Mukherjee [25] formulated an inventory model where a two-parameter Weibull distribution has represented the rate of deterioration. Further, Chan et al. [26] framed an integrated production inventory model for exponentially deteriorating items considered the products' deterioration during delivery. Banerjee & Agrawal [27] studied the optimal discounting and the ordering policies in an inventory system considering the general deterioration distribution. Khakzad & Gholamian [28] constructed an inventory model for deteriorating products that studied the effect of inspection times during the replenishment period on the deterioration rate. When we model an inventory or supply chain problem, we consider various factors associated with the system. All the strategies aim to minimize the total cost of the inventory or the supply chain system [29], [30], [31].

In this study, we have assumed that the manufacturer starts producing the items at a lower rate to avoid a large stock of products at the initial stage. He switches on to a higher production rate when the inventory level reaches a prefixed level. This strategy may help him to get rid of the holding cost and the deterioration cost of the products and consequently the total cost of the production system. Furthermore, starting at a higher

rate may block the manufacturer's investment in production for a longer period, then losing the opportunity to earn interest from it. Again, inflation and the time value of money are the other essential factors impacting the economy significantly, and nowadays, it is becoming a permanent feature of the financial system. Therefore, it is crucial to consider this factor while developing a production inventory model. Best known to the authors of this paper, no author considered these issues in their studies. Although two studies by Roy and Samanta [5] and Bhowmick and Samanta [6] considered the two production rates, they did not consider the effect of inflation and the time value of money. Therefore, to address this evidence gap, this study aims to determine the cost minimization policy of a manufacturer producing items that deteriorate over time considering all these issues. We derived the optimum production time by minimizing the total cost of the system. We illustrated the applicability of the proposed model numerically using LINGO 18.0 software. Also, we studied the effect of changes in the parameters on the optimum decision variables. This model may help the manufacturing companies to make production decisions to optimize the total cost of the system.

This paper has been organized in the following sequence. Section 2 presented the notations, assumptions used to formulate the model, mathematical modeling, the solution methodology, numerical examples, and the sensitivity analysis procedure. In addition to that, the results and discussion has been carried out in section 3. Finally, we concluded section 4.

2. Methods

2.2. Notations and Assumptions

We used the following notations to formulate the model

$I(t)$: Inventory level of the manufacturer at time t.

a : constant demand rate

p_1 : production rates started at time $t = 0$

t_1 : Time up to which the production is running at a lower rate p_1

p_2 : production rates started at time $t = t_1$

t_2 : Time up to which the production is running at a lower rate p_2

θ : constant rate of deterioration.

Q_1 : Inventory level at time $t = t_1$

S : Inventory level at time $t = t_2$

C_m : Production cost per unit for the manufacturer

h_m : Holding cost per unit time

d_m : Deterioration cost per unit

A_m : Setup cost per cycle

r : $\delta - i$, where δ is the interest rate per unit of currency, and i is the inflation per unit currency.

T : The fixed duration of a production cycle.

SC : Total set-up cost

PC : Total production cost

HC : Total holding cost

DC : Total deterioration cost

H : The determinant value of the Hessian matrix

TC_m : Total cost of the manufacturer

We have formulated this model based on the following assumptions:
  i.   Demand rate is known, constant, and continuous during the planning horizon under consideration.
  ii. There is no repair of deteriorating items during the period.
  iii. The production system produces a single item.
  iv. The planning horizon is finite.
  v.   In this model, we have considered the effect of inflation and the time value of money.
  vi. Shortages are not allowed.
  vii. The rate of deterioration is constant.

The production of the item is started at time $t = 0$, at a production rate p_1 and continues up to time $t = t_1$, when the inventory level reaches Q_1, then the rate of production is switched over to a higher rate p_2 and the production is stopped when the inventory level reaches S. The inventory depletes due to constant demand (a) and deterioration (θ) and reaches zero level at time $t = T$.
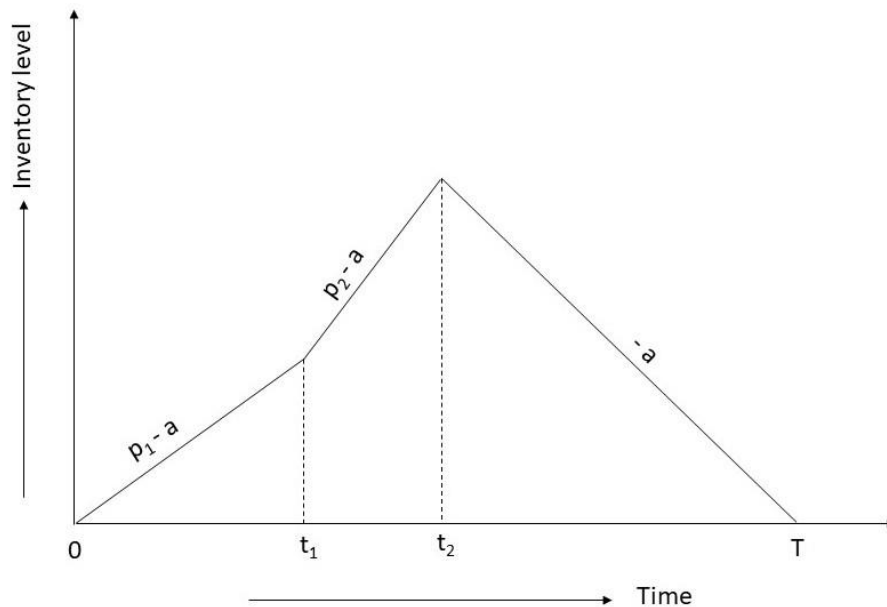

Fig. 1 Pictorial Representation of the Model

2.2. Mathematical Modeling

In this section, the mathematical model to derive the cost function of the manufacturer is constructed. The manufacturer starts producing the items at a lower rate p1 at time t = 0 and continues up to time t = t1 when his inventory level reaches a prefixed level Q1. He switches on to a higher production rate p2 at time t = t1 and keeps his manufacturing process up to time t = t2. At this point, his inventory level reaches the highest level S. Afterwards, his inventory level depletes due to customers' demand and the products' deterioration and reaches the zero level at time t = T. Fig. 1 represents the pictorial representation of this model.

Let $I(t)$ be the on-hand inventory at time t ($0 \leq t \leq T$). Then the differential equations describing the inventory level $I(t)$ in the interval $[0, T]$ are given by the following construct:

$$\frac{dI(t)}{dt} + \theta\, I(t) = \begin{cases} p_1 - a, & 0 \leq t \leq t_1 \\ p_2 - a, & t1 \leq t \leq t_2 \\ -a, & t_2 \leq t \leq T \end{cases} \tag{1}$$

With boundary conditions:

$$I(0) = 0, I(t_1) = Q_1, I(t_2) = S, I(T) = 0 \tag{2}$$

The solutions of the equations (1) are given by

$$I(t) = \begin{cases} \frac{1}{\theta}(p_1 - a)(1 - e^{-\theta t}), & 0 \leq t \leq t_1 \\ \frac{1}{\theta}(p_2 - a) + \left[Q_1 - \frac{1}{\theta}(p_2 - a)\right] e^{-\theta(t-t_1)}, & t_1 \leq t \leq t_2 \\ -\frac{1}{\theta}a + (S + \frac{1}{\theta}a)e^{-\theta(t-t_2)}, & t_2 \leq t \leq T \end{cases} \tag{3}$$

Using the boundary condition $I(t_1) = Q_1$, from (3) we have

$$Q_1 = \frac{1}{\theta}(p_1 - a)\left(1 - e^{-\theta t_1}\right) \tag{4}$$

Again, from the boundary condition $I(t_2) = S$, from (3) we have

$$0 = -\frac{a}{\theta} + \left(S + \frac{a}{\theta}\right)e^{-\theta(T-t_2)}$$

i.e.,

$$S = \frac{a}{\theta}\left(e^{\theta(T-t_2)} - 1\right) \tag{5}$$

Now present worth setup cost

$$SC = A_m \tag{6}$$

Present worth production cost

$$PC = C_m\left[p_1 \int_0^{t_1} e^{-rt} dt + p_2 \int_{t_1}^{t_2} e^{-rt} dt\right]$$

$$= C_m(-\frac{p_1}{r}(e^{-rt_1} - 1) - \frac{p_2}{r}(e^{-rt_2} - e^{-rt_1}))$$

$$= C_m p_1\left(t_1 - \frac{rt_1^2}{2}\right) + C_m p_2\left\{(t_2 - t_1 - \frac{r}{2}(t_2^2 - t_1^2)\right\} \tag{7}$$

Present worth holding cost:

$$HC = h_m \left[\int_0^{t_1} I(t)e^{-rt}\, dt + \int_{t_1}^{t_2} I(t)e^{-rt} dt + \int_{t_2}^{T} I(t)e^{-rt}\, dt \right]$$

$$= h_m \left[(p_1 - a)\frac{t_1^2}{2} + (p_1 - a)t_1 \left\{ (t_2 - t_1) - \frac{r}{2}(t_2^2 - t_1^2) - \frac{\theta}{2}(t_2 - t_1)^2 \right\} \right.$$

$$+ \frac{1}{2}(p_2 - a)(t_2 - t_1)^2 + a\, T \left\{ -\frac{(r+\theta)}{2}(T^2 - t_2^2) + (T - t_2)^2 \right\} \tag{8}$$

$$\left. -\frac{a}{2}(T^2 - t_2^2) \right]$$

Cost of deteriorated units:

$$DC = d_m\theta \left[\int_0^{t_1} I(t)e^{-rt}\, dt + \int_{t_1}^{t_2} I(t)e^{-rt}\, dt + \int_{t_2}^{T} I(t)e^{-rt}\, dt \right]$$

$$= d_m\theta \left[(p_1 - a)\frac{t_1^2}{2} + (p_1 - a)t_1 \left\{ (t_2 - t_1) - \frac{r}{2}(t_2^2 - t_1^2) - \frac{\theta}{2}(t_2 - t_1)^2 \right\} \right.$$

$$+ \frac{1}{2}(p_2 - a)(t_2 - t_1)^2 + a\, T \left\{ -\frac{(r+\theta)}{2}(T^2 - t_2^2) + (T - t_2)^2 \right\} \tag{9}$$

$$\left. -\frac{a}{2}(T^2 - t_2^2) \right]$$

Present worth total cost:

$$TC_m = SC + PC + HC + DC$$

$$= A_m + C_m p_1 \left(t_1 - \frac{rt_1^2}{2} \right) + C_m p_2 \left\{ (t_2 - t_1) - \frac{r}{2}(t_2^2 - t_1^2) \right\}$$

$$+ (h_m + d_m\theta) \left[(p_1 - a)\frac{t_1^2}{2} \right. \tag{10}$$

$$+ (p_1 - a)t_1 \left\{ (t_2 - t_1) - \frac{r}{2}(t_2^2 - t_1^2) - \frac{\theta}{2}(t_2 - t_1)^2 \right\}$$

$$+ \frac{1}{2}(p_2 - a)(t_2 - t_1)^2 + a\, T \left\{ -\frac{(r+\theta)}{2}(T^2 - t_2^2) + (T - t_2) \right\}$$

$$\left. -\frac{a}{2}(T^2 - t_2^2) \right]$$

Lemma. The cost function(TC_m) of the manufacturer is convex with respect to t_1.

The cost function TCm will be convex with respect to t1 if its second derivative is greater than zero. Therefore, to examine the convexity of the cost function TCm in equation (10), differentiating it twice with respect to t_1 we have –

$$\frac{\partial(TC_m)}{\partial t_1} = C_m p_1(1 - rt_1) + C_m p_2(-1 + rt_1)$$
$$+ (h_m + d_m\theta)\Big[(p_1 - a)t_1$$
$$+ (p_1 - a)\Big\{(t_2 - t_1) - \frac{r}{2}(t_2{}^2 - t_1{}^2) - \frac{\theta}{2}(t_2 - t_1)^2\Big\}$$
$$+ (p_1 - a)t_1\{-1 + rt_1 + \theta(t_2 - t_1)\} - (p_2 - a)(t_2 - t_1)\Big] \quad (11)$$

And,

$$\frac{\partial^2(TC_m)}{\partial t_1{}^2} = rC_m(p_2 - p_1) + (h_m + d_m\theta)\,[2(p_1 - a)\{rt_1 + \theta(t_2 - t_1)\} + (p_2 - p_1)$$
$$+ (p_1 - a)t_1(r - \theta)] \quad (12)$$

Since $p_2 > p_1$, $p_1 > a$, $p_2 > a$ and $t_2 > t_1$, $\frac{\partial^2(TC_m)}{\partial t_1{}^2} > 0$.

Therefore, TC_m is convex with respect to t_1.

2.3. Solution Methodology

In this model, the decision variables are the time t1 and t2. The manufacturer should continue the production at a lower rate p1 and the higher rate p2, respectively. Therefore, our purpose is to derive the optimum values of t1 and t2, keeping the total cost of the manufacturer at an optimum (minimum) level. It can be achieved from the necessary condition to minimize the total cost TCm.

Now, the necessary conditions to minimize the cost TCm are

$$\frac{\partial(TC_m)}{\partial t_1} = 0 \text{ and } \frac{\partial(TC_m)}{\partial t_2} = 0$$

Therefore, we can get the optimum value of t1 by equating the right-hand side of equation (11) and that of t2 from the following equation

$$\frac{\partial(TC_m)}{\partial t_2} = C_m p_2(1 - rt_2)$$
$$+ (h_m + \theta d_m)[(p_1 - a)t_1\{1 - rt_2 - \theta(t_2 - t_1)\} + (p_2 - a)(t_2 - t_1)$$
$$+ aT\{(r + \theta)t_2 - 1\} + at_2] = 0 \quad (13)$$

The values of t1 and t2 obtained will optimize the total cost of the manufacturer if these values of t1 and t2 satisfy the sufficient conditions –

$\frac{\partial^2(TC_m)}{\partial t_1{}^2} > 0$ and the determinant value of the Hessian matrix (H) –

$$\begin{bmatrix} \dfrac{\partial^2(TC_m)}{\partial t_1{}^2} & \dfrac{\partial^2(TC_m)}{\partial t_1\partial t_2} \\ \dfrac{\partial^2(TC_m)}{\partial t_1\partial t_2} & \dfrac{\partial^2(TC_m)}{\partial t_2{}^2} \end{bmatrix} > 0.$$

i.e., if

$$H = \frac{\partial^2(TC_m)}{\partial t_1{}^2}\cdot\frac{\partial^2(TC_m)}{\partial t_2{}^2} - \left(\frac{\partial^2(TC_m)}{\partial t_1\partial t_2}\right)^2 > 0$$

Now,

$$\frac{\partial^2(TC_m)}{\partial t_2{}^2} = -rC_mp_2 + (h_m + \theta d_m)\{p_2 + aT(r + \theta) - (p_1 - a)(r + \theta)t_1\} \tag{14}$$

And,

$$\frac{\partial^2(TC_m)}{\partial t_1\partial t_2} = (h_m + \theta d_m)\{(p_1 - p_2) - (p_1 - a)(r + \theta)t_2 + 2(p_1 - a)\theta t_1\} \tag{15}$$

Therefore, using the equations (12), (14), and (15) we get,

$$\begin{aligned} H = \frac{\partial^2(TC_m)}{\partial t_1{}^2}\cdot\frac{\partial^2(TC_m)}{\partial t_2{}^2} - \left(\frac{\partial^2(TC_m)}{\partial t_1\partial t_2}\right)^2 &= rC_m(p_2 - p_1) + (h_m \\ &+ d_m\theta)\,[2(p_1- a)\{rt_1 + \theta(t_2 - t_1)\} + (p_2 - p_1) \\ &+ (p_1- a)t_1(r - \theta)].\,[-rC_mp_2 \\ &+ (h_m + \theta d_m)\{p_2 + aT(r + \theta) \\ &- (p_1 - a)(r + \theta)t_1\}] \\ &- [(h_m + \theta d_m)\{(p_1 - p_2) - (p_1 - a)(r + \theta)t_2 \\ &+ 2(p_1 - a)\theta t_1\}]^2 \end{aligned} \tag{16}$$

We will prove the sufficient condition (H> 0) numerically. Using equation (11) and equation (13), we calculated the optimum values of t1 and t2 and the corresponding total cost of the manufacturer in the numerical example section. If the solutions obtained from equations (11) and (13) do not meet sufficient conditions, we may conclude that no feasible solution will be optimal for the set of parameter values. Such a situation will imply that the parameter values are inconsistent, and there are some errors in their estimation. We used LINGO 18.0 software for calculation.

2.4. Numerical Example

Example-1: We consider the following numerical values of the parameters in appropriate units to analyze the model

A_m=150 \$, C_m=10 \$/unit, h_m=2 \$/unit, d_m=7 \$/unit, p_1=75 unit per unit of time, p_2=100 unit per unit of time, δ=0.7, i=0.3, θ =0.08, T =4 unit of time and a=70 unit per unit time. Then we have the values of t_1, t_2, Q_1, S and TC_m are respectively t_1=2.57 unit of time, t_2 =3.22 unit of time, Q_1 =11.8 unit, S = 56.05 unit and TC_m =932.11\$ in appropriate units. Using these values, we get the determinant value of the Hessian matrix H = 31087.20 >0. Therefore, these results are optimum.

In **Table 1**, we have shown the change of the values of t_1, t_2, Q_1, S and TC_m with the change of the parameters θ, r, C_m, p_1 and p_2.

In **example 1**, if we took $p_1 = p_2 = 75$ unit per unit of time, we found that the total cost of the system as TC_m =937.09 \$. Again, if we set $p_1 = p_2 = 100$ unit per unit time, we found that the total cost of the inventory system was as TC_m =1320.20 \$. Also, if $p_1 = 100$ and $p_2 = 90$ unit per unit of time, then the total cost was TC_m =1335.37 \$.

Example-2: Consider another inventory system with the following data-

a_m=200 \$, C_m=20 \$/unit, h_m=5 \$/unit, d_m=16   \$/unit, p_1=175   unit/day, p_2=200 unit/day, δ=0.7, i=0.3, θ=0.08/day, T=5 day , a=150 unit.

Then we have the values of t_1, t_2, Q_1, S and TC_m are projected respectively as t_1=2.65 days, t_2= 4.03 days, $Q_1 = 59.72$ unit, S = 151.29 unit and TC_m= 2138 \$. Here, H = 1317732 >0, so, these results are optimum.

2.5. Sensitivity Analysis

In this section, we carried out a sensitivity analysis to investigate the effect of changes in some parameters θ, i, C_m, p_1 and p_2 associated with the model on the optimum decision variables. In practice, the deterioration, inflation, and production cost were found to have an adverse effect on the business economy. Furthermore, in any manufacturing system, the manufacturer needs to fix the production rate and know how the production rate changes impact the production system's cost. Therefore, it is essential to perform this sensitivity analysis. The analysis was done by changing one parameter at a time, keeping the other parameters of the system unchanged.

3. Results and Discussion

In the numerical example subsection, we derived the optimum values of the decision variables and the system's total cost considering different production rates. Comparing all these results, it was concluded that the total cost would be minimum if the production unit started producing the items at a lower production rate and switched over to a higher production rate when the inventory level reached a prefixed level. Therefore, we found the production assumption of this model economic for the manufacturer.

This analysis was done based on example 1. The results of this sensitivity analysis are presented in Table 1. Based on Table 1, we observed that the production time increased slightly with the increase in the deterioration rate (θ), but the total cost decreased. Similar to the study by Bhowmick and Samanta [6], this study's findings signified a decrease in the stock levels Q1 and S with the increase in the deterioration rate. Also, the total cost of the system was susceptible towards the rate of inflation r. The production time and the total cost both decreased with the reduction of the inflation rate. This result is quite apparent, and it is similar to the finding of the studies [16], [17], [18]. Furthermore, with the increase in the production cost, the production time decreased. However, the total cost of the system increased, resembling the findings of the study [6]. Moreover, with the increase in the production rate p_1, the production time at this rate decreased slightly. However, the total cost increased significantly (which is also similar to the study by Bhowmick and Samanta [6]. Therefore, it is wise for the manufacturer to keep the initial production rate as lower as possible. Again, with the increase in the production rate p_2, the production time and the total cost remain almost static. All in all, the producer can increase the later production rate as higher as needed.

Table 1 Changes in t_1, t_2, Q_1, S and TC_m with the changes of the parameters $\theta, r,$ C_m, p_1 and p_2.

Parameter	Parameter value	t_1	t_2	Q_1	S	TC_m
θ	0.06	2.60	3.19	12.05	57.84	936.10
	0.07	2.61	3.21	11.92	56.94	934.05
	0.08	2.62	3.22	11.80	56.05	932.11
	0.09	2.62	3.24	11.68	55.18	930.28
	0.1	2.63	3.25	11.55	54.33	928.56
i	0.30	2.62	3.22	11.80	56.05	932.11
	0.25	2.40	3.08	10.90	67.12	697.37
	0.20	2.22	2.96	10.15	76.03	469.96
	0.15	2.06	2.86	9.51	83.38	247.45
	0.10	1.93	2.78	8.95	89.56	28.38
C_m	8	2.67	3.32	12.03	48.54	768.32
	10	2.62	3.22	11.80	56.05	932.11
	12	2.58	3.14	11.64	62.14	1101.03
	14	2.55	3.08	11.52	67.14	1273.83
	16	2.53	3.02	11.45	71.30	1449.34
p_1	70	2.78	3.22	0.00	56.23	827.13
	75	2.62	3.22	11.80	56.05	932.11
	80	2.46	3.23	22.31	55.88	1031.75
	85	2.30	3.23	31.55	55.67	1126.35
	90	2.15	3.23	39.47	55.36	1216.04
p_2	90	2.55	3.25	11.52	53.89	937.41
	95	2.59	3.24	11.69	54.99	934.82
	100	2.62	3.22	11.80	56.05	932.11
	105	2.63	3.21	11.87	57.07	929.37
	110	2.64	3.20	11.92	58.06	926.65

4. Conclusion

In this paper, we have developed a production inventory model for deteriorating items considering the effect of inflation and the time value of money. The initial production rate of the manufacturer was low. However, it switched to a higher production rate at the time when the stock level of the produced commodities reached a prefixed level. We found this strategy economical for the manufacturer. This model might help managers of various industries make production decisions wisely to minimize their production system's total cost. This model can be extended in several ways. Most importantly, researchers can consider the trade credit policy or the manufacturer's quantity discount. Moreover, they may consider various types of production and demand rates.

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