Optimization Multi-Item Lot Sizing Model involve Transportation and Capacity Constraint under Stochastic Demand using Aquila Optimizer

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ABSTRACT

Inventory issues are often a major concern as they significantly impact operating costs. Lot sizing is one of the key decisions in managing inventory. However, in a real context, the demand for each item is often uncertain. It is supplied from the same supplier, requiring product orders to be placed in the same period. In addition, limited vehicle capacity and transportation costs are important factors to consider in making multi-item lot sizing decisions. The purpose of this study is to propose a new inventory model multi-item lot sizing model involving transportation cost and capacity constraints under stochastic demand. The decision variables involved in the model are each item's ordering cycle and safety factor with the objective function of minimizing the total inventory cost. To optimize the inventory model, this study also offers the advanced procedure of the Aquila Algorithm. This study also presents sensitivity analysis to the appropriate policy for optimizing the multi-item lot-sizing inventory problem involving transportation cost and capacity constraint under stochastic demand.

1. Introduction

More manufacturing industries are growing and developing due to good inventory management [1]. Inventory is the management of goods from raw materials to a product [2, 3]. Inventory is a vital component in the production chain, where efficiency in procurement, inventory control, and logistics greatly affects the smooth production process and company profitability [4, 5]. One of the key decisions in managing inventory is the lot sizing decision. There are two lot-size models in inventory: single-item and multi-item [6]. Single-item lot sizing is an inventory planning method for one product type. In contrast, multi-item lot sizing is an inventory planning method used for several types of products [7]. In lot sizing, ordering optimization is largely based on minimizing the total cost of inventory and generating maximum profit [1, 8, 9]. Nowadays, the increasingly diverse types of raw materials and the presence of factors such as spikes in transportation costs and unpredictable demand fluctuations present complexities in inventory management

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[10, 11]. To overcome these challenges, the Multi-Item Lot Sizing model can effectively optimize raw material inventory management.

Previous research has presented various Multi-Item Lot Sizing inventory models. Harris [12] initially developed Economic Order Quantity (EOQ) to determine the optimal number of lot sizes for periodic product purchases with minimal procurement costs. Chakrabarty, et al. [13] extended EOQ to address two limited storage warehouses, backordering, and financial issues. Mirzazadeh, et al. [14] offered a Multi-Item inventory model with budget and discount considerations. Đorđević, et al. [15] introduced Multi-Item EOQ with per-unit purchase cost and warehouse capacity constraints. Meanwhile, Silitonga, et al. [16] extended the Multi-Item inventory model with probabilistic demand, raw material expiry, purchase discounts, and warehouse capacity constraints. Silitonga and Sembiring [17] integrated time delay and warehouse capacity constraints in the Multi-Item EOQ method. Ghafour, et al. [18] considered investment costs and warehouse capacity in Multi-Item EOQ. Rossi, et al. [19] focused on Multi-Item EOQ with warehouse capacity constraints. Meanwhile, Pal, et al. [20] related demand to product selling price and price reduction rate in the conventional EOQ method. Cárdenas-Barrón and Sana [21] discussed an EOQ model with variable demand and two suppliers. Sicilia Rodríguez, et al. [22] developed a Multi-Item EOQ method by involving stochastic demand, allowing backorders, and considering available warehouse constraints. Fallahi, et al. [23] discussed Multi-Item EOQ for reusable products in their research. This previous research has produced a variety of methods that are relevant in optimizing multi-item inventory management. Unfortunately, most of these studies assume that demand characteristics are deterministic.

Researchers also offer several metaheuristic procedures to optimize multi-item lot sizing. Khalilpourazari, et al. [24] developed Multi-Item EOQ by providing constraints on warehouse capacity and raw material purchase budget. It was solved using the Water Cycle Algorithm and Whale Optimization Algorithm (WCWOA) procedures. In addition, Khalilpourazari and Pasandideh [25] optimized the Multi-Item EOQ model by involving the number of defective items using the Sine Cosine Crow Search Algorithm. In addition, other research conducted by Pasandideh, et al. [26] proposed a Genetic Algorithm (GA) procedure for Multi-Item EOQ optimization with warehouse capacity constraints. Particle swarm optimization (PSO) was also proposed by Fallahi, et al. [23] for multi-item lot-sizing optimization. Unfortunately, most studies assume that each item is supplied from different suppliers. Very few studies assume that multiple items are supplied from the same supplier. In addition, previous studies also assumed that the vehicle capacity is infinite. However, studies that limit vehicle capacity have received less attention. Most studies ignore the transportation fuel consumption due to the delivery activity of the order. Fuel consumption in transportation plays a key role in inventory management. Companies can reduce transportation costs by making fuel consumption efficient and increasing profits [27]. In addition, fuel consumption impacts greenhouse gas emissions, and companies can reduce their carbon footprint by reducing fuel usage [28].

In addition, most studies utilize heuristic procedures that do not guarantee an optimal solution [14, 15, 20, 21]. In inventory model optimization, no study utilizes the Aquila algorithm for Multi-Item Lot Sizing optimization. The Aquila algorithm is an optimization procedure inspired by the behavior of Aquila offered by Abualigah, et al. [29]. This algorithm is proven to be able to optimize various problems such as scheduling [30], forecasting [31, 32], and engineering optimization [33]. Based on these advantages, this study proposes the Aquila Algorithm procedure for optimizing Multi-Item inventory problems involving transportation costs, vehicle capacity, and stochastic demand to minimize total inventory costs. Based on this description, this study proposes a new Multi-

Item Lot Sizing model involving Transportation Costs and vehicle capacity constraints to deal with Stochastic Demand. In addition, this study proposes a new procedure, the Aquila algorithm, for inventory optimization. This research has significant contributions to multi-item inventory management and inventory optimization. With the main objective to propose a Multi-Item Lot Sizing model that considers Transportation Cost and vehicle capacity constraints in the face of Stochastic Demand, this research bridges the gap in the existing literature. Moreover, by proposing a new procedure of Aquila algorithm for inventory optimization algorithm, this research offers an innovative approach to improve inventory management efficiency.

2. Methods

2.1 System Characteristics

This section describes the characteristics of the multi-item lot sizing model. In the proposed model, the demand for each product item is stochastic based on a normal distribution with a known standard deviation and average demand. Since each product has a different standard deviation and average demand, the decision maker must determine the appropriate safety factor to optimize the inventory of each item. Each product item is supplied from the same supplier so that the product's ordering cycle (T^*) is the same. For each order, the total quantity of each item should not exceed the vehicle capacity. Furthermore, the objective function of this problem is to minimize the total inventory cost. The illustration of this multi-item lot-sizing problem is illustrated in Fig. 1. Because of the purchase of raw materials from the same supplier, the ordering cycle (T^*) is the same for each product item. However, the order quantity value (Q_i) is different for each item. The order quantity for each item can be calculated based on $Q_i = D_i T^*$

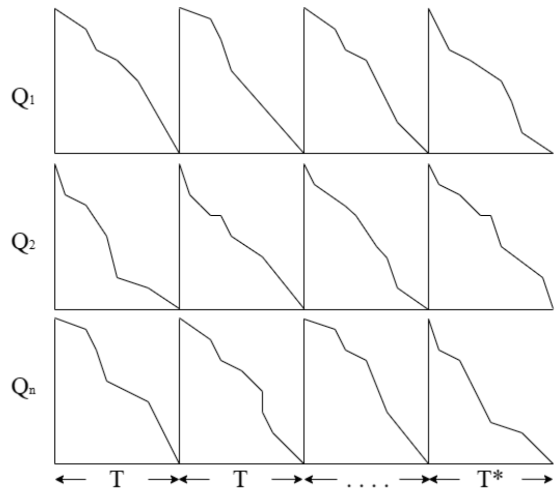

Fig. 1. Order cycle

2.2 Assumptions and Notations

This section presents the assumptions and notations used in the model. The assumptions used in the proposed model are (1) Demand for each item is normally distributed with known standard deviation and average; (2) Each item is supplied from the same supplier; (3) vehicle capacity is limited; (4) Backorders are not allowed; and (5) delivery lead time is ignored.

Meanwhile, the notations used in the proposed model are described as follows:

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i : $1,2,3,\ldots,n$
n : Total item
C_{is} : One-time cost of ordering from the same supplier (Rp)
P_i : Purchase cost of item i (Rp)
H_i : Product inventory cost i (Rp)
L_i : Lost sale cost of product i (Rp)
D_i : Product demand i (unit/moon)
Q_i : Order quantity of item i (unit)
T^* : Order cycle with the same supplier
k_i : Safety factor item i
σ_i : Standard deviation of product i
$f_s(k)$: Probability density function of standard normal distribution
$F_s(k)$: Cumulative function of standard normal distribution
a : Transportation fixed costs (Rp)
FL : Fuel consumption (km/liter)
j : Delivery mileage (km)
β : Vehicle fuel prices (Rp)
p : 2%
M : 50 kg
V_{cap} : Vehicle capacity
ETC : Expected total inventory cost (Rp)
SS_i : Safety stock (unit)
ES_i : Estimated loss (unit)
C_o : Order cost (Rp)
C_p : Purchase cost (Rp)
C_t : Transportation cost (Rp)
C_h : Inventory cost (Rp)
C_L : Loss sale cost (Rp)

2.3 Proposed Model

This section presents the mathematical model of the proposed Multi-Item Lot Sizing Model to minimize the total inventory cost (TC). The proposed model is developed based on the research of Đorđević, et al. [15]. Not only that, in this study, stochastic demand behavior is adopted from Utama, et al. [11]. This research involves transportation costs because in the process of purchasing raw materials, one of the highest costs is transportation costs developed based on research conducted by Utama, et al. [34] and Utama, et al. [35], which discusses fuel consumption and vehicle capacity.

In the proposed model, the order cost is one of the cost components used. Order cost is spent in one order or as a preparation cost during ordering. Equation (1) is a formula for the cycle time of ordering raw material items. Meanwhile, the total message cost can be calculated using Equation (2).

$$T^* = \frac{Q_1}{D_1} = \frac{Q_2}{D_2} = \frac{Q_n}{D_n} \quad (1)$$

$$C_o = \frac{C_{is}}{T^*} \quad (2)$$

The next cost component is the purchase cost. The total purchase cost is the total cost incurred to purchase the items ordered according to the needs within the company. The calculation of total purchase costs can be calculated based on Equation (3).

$$C_p = \sum_{i=1}^{n} P_i D_i \tag{3}$$

Another cost component is transportation costs. Total transportation costs are the costs incurred by the company to send goods from suppliers to the company. Transportation costs are calculated based on driver fees and vehicle fuel. The delivery of raw materials has two transportation cycles: when the vehicle is unloaded and loaded. Therefore, the total transportation cost formula can be seen in Equation (4).

$$C_t = a + \left[\frac{j}{FL} \times \left(1 + p \times \left[\frac{\sum_1^n B_i.D_i T^*}{M}\right]\right)\right] \beta \times \frac{1}{T^*} \tag{4}$$

In addition, what is considered in this study is stochastic demand that is normally distributed. To anticipate uncertain demand, safety stock is required. The safety stock value corresponds to the standard deviation of demand and the safety factor used, formulated in Equation (5). Meanwhile, the total storage cost is the total cost required during the storage process for all items. The total storage cost can be calculated based on Equation (6).

$$SS_i = k_i \sigma_i \sqrt{T^*} \tag{5}$$

$$C_h = \sum_{i=1}^{n} H_i \left(\frac{D_i T^*}{2} + SS_i\right) \tag{6}$$

Not only considering safety stock, but this research also considers the amount of lost sales due to the inability to meet demand. Expected lost sales are formulated in Equations (7) and (8). The total estimated cost of lost sales can be calculated in Equation (9).

$$ES_i = \sigma_i \sqrt{T^*} \psi(k) \tag{7}$$

$$\psi(k) = \{f_s(k) - k\,[1 - F_s(k)]\} \tag{8}$$

$$C_L = \left(\frac{1}{T}\right) L_i ES_i \tag{9}$$

Based on these components, the total inventory cost can be calculated by adding up the costs of ordering, purchasing, transportation, storage, and loss. Expected Total inventory cost is formulated in Equation (10).

$$ETC = \frac{C_{is}}{T^*} + \sum_{i=1}^{n} P_i D_i + (a + \left[\frac{j}{FL} \times \left(1 + p \times \left[\frac{\sum_1^n B_i.D_i T^*}{M}\right]\right)\right] \beta \times \frac{1}{T^*}) + \sum_{i=1}^{n} H_i \left(\frac{D_i T^*}{2} + k_i \sigma_i \sqrt{T^*}\right) + \sum_{i=1}^{n} \left(\frac{1}{T^*}\right) L_i ES_i \tag{10}$$

Therefore, based on the total cost expectation formula, the inventory system can be formulated with a Non-Linear Programming model based on the following Equation (11)-(14):

$$Min\ ETC = C_o + C_p + C_t + C_h + C_L \tag{11}$$

Constraints:

$$0 < T^* \leq 1 \tag{12}$$

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$$0 < k_i \leq 2.99 \tag{13}$$

$$\sum_{i=1}^{n} D_i T^* \leq V_{cap} \tag{14}$$

Equation (11) shows the expected total inventory cost of the model discussed in this study that must be minimized. This optimization must meet several constraints. The ordering cycle time must be greater than 0 and not exceed 1 as shown in Equation (12). Meanwhile, the constraint in Equation (13) illustrates that the safety factor of each item (k_i) in the normal distribution must be greater than 0 and cannot exceed 2.99. Finally, the constraint in Equation (14) is used to guarantee that the total order quantity of each item should not exceed the vehicle capacity. Minimize the inventory system's total cost, which can be achieved by simultaneously determining the optimal decision variables, namely the ordering cycle (T^*) and the safety factor of each item (k_i).

2.4 Aquila Algorithm

This section presents the proposed procedure for optimizing the proposed inventory model. The Aquila population-based optimization method (X) is stochastically generated between the upper and lower bounds. Equation (15) is used to calculate the population of the Aquila flock. Based on Equation (16), the position vector of each aquila is randomly generated. The total aquila candidate solution (population) is represented by N, and Dim represents the dimensional size of the solution. Where X_N, Dim denotes the position vector of Aquila N in the solution dimension Dim. Rand denotes a random number in the range 0-1. The upper bound of the given problem is U_b and the lower bound is L_b. Dim is the number of tasks to be solved.

$$X = \begin{bmatrix} X_{1,1} & \cdots & X_{1,j} & X_{1,Dim-1} & X_{1,Dim} \\ X_{2,1} & \cdots & X_{2,j} & \cdots & X_{2,Dim} \\ \cdots & \cdots & X_{3,j} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X_{N-1,1} & \cdots & X_{N-1,j} & \cdots & X_{N-1,Dim} \\ X_{N,1} & \cdots & X_{N,j} & X_{N,Dim-1} & X_{N,Dim} \end{bmatrix} \tag{15}$$

$$X_{ij} = randx(U_b - L_b) + L_b, \qquad i = 1,2, \dots, N_j = 1,2, \dots, Dim \tag{16}$$

Aquila can spot their prey from a high vantage point, which helps them choose the optimal hunting territory. A flock of Aquila foraging at high altitudes will make decisions regarding the search area at this location. Equation (17) is a mathematical model formulation describing vertical humpbacks' soaring flight behavior.

$$X_1(t+1) = X_{best}(t) \times \left(1 - \frac{t}{T}\right) + (X_M(t) - X_{best}(t) * rand) \tag{17}$$

The solution of the next iteration is formulated as $X_1(t+1)$, the result of the high soar method with vertical bending. The best solution symbol denotes the prey location denoted as $X_{best}(t)$ used in the t-th iteration. Equation $\left(1 - \frac{t}{T}\right)$. is used to model the extended search control (exploration) for each iteration. The average value of the solution location at iteration t is called $XM(t)$, which can be obtained using Equation (18). The current and maximum iteration symbols are t and T, and a rand is a random number between (0, 1). The number of decision variables is dim, and N is the number of possible solutions.

$$X_M(t) = \frac{1}{N}\sum_{t=1}^{N} X_i(t),\ \forall_j = 1,1,\dots,Dim \tag{18}$$

The Aquila circles above its prey and prepares the terrain before attacking after locating the prey area from a high altitude. The behavioral name of this method is contour flying with a short gliding attack. The Aquila carefully inspects the prey's target territory before attacking. Equation (19) describes this action mathematically.

$$X_2(t+1) = X_{best}(t) + Levy(D) + X_R(t) + (y-x) * rand \tag{19}$$

$X_2(t+1)$ is the solution of the next iteration generated by the contour flight with a short glide attack. D denotes the dimensional space, and Levy (D) is the distribution function of the retribution flight. Levy (D) can be mathematically calculated based on Equation (20). XR(t) is a random solution taken in the range of (1 N) during the t-th iteration. u and v are random numbers ranging between 0 and 1. The value of the constant variable s is 0.01. The value of σ is found using Equation (21).

$$Levy\ (D) = s \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}} \tag{20}$$

$$\sigma = \left(\frac{r(1+\beta) \times sine\left(\frac{\pi\beta}{2}\right)}{r\left(\frac{1+\beta}{2}\right) \times \beta \times 2\left(\frac{\beta-1}{2}\right)} \right) \tag{21}$$

For the variable β, the constant fixed value is 1.5. The y and x values found in Equations (22) and (23) indicate the spiral shape in prey search. These values are calculated using equations 24 and (6. The value of r_1 ranges between 1 and 20. The constant variable U is 0.00565, the constant variable ω is 0.005, and D_1 is a representation of integers from 1 to Dim.

$$y = r \times \cos(\theta) \tag{22}$$

$$x = r \times \sin(\theta) \tag{23}$$

$$r = r_1 + U \times D_1 \tag{24}$$

$$\theta = -\omega \times D_1 + \theta_1 \tag{25}$$

$$\theta_1 = \frac{3 \times \pi}{2} \tag{26}$$

Aquila makes an initial attack by descending vertically at the exploitation stage (X_3) to see the reaction of its prey. A low flight technique with a slow descent attack is the behavior. Here, the Aquila attacks the prey by using a set target area. Equation (27) is the numerical model for the behavior of low flight with slow descent attack. The solution of the next t iteration of the low-flight method with slow descent attack is represented as $X_3(t+1)$), and $X_M(t)$ is the average value of the $t-th$ iteration solution modeled in Equation (20). $X_{best}(t)$ is the best solution obtained up to the $t-th$ iteration, which indicates the approximate location of the prey. The exploitation adjustment parameters a and δ are displayed between the values (0,1). $rand$ denotes a random value with a range of (0,1), U_b and L_b are the upper and lower bounds of the given problem.

$$X_3(t+1) = \left(X_{best}(t) - X_M(t)\right) \times a - rand + \left((U_b - L_b) \times rand + L_b\right) \times \delta \tag{27}$$

Algorithm 1 Aquila Algorithm

```
Initialization phase :
Initialize the population X of the GEO
Initialize the  parameters of the GEO (i.e., α, δ, etc).
while (The end condition is not met) do
    Calculate the fitness function values
    Xbest(t)=Determine the best obtained solution according to the fitness values.
    for (i=1,2,...,N) do
        Update the mean value of the current solution XM(t)
        Update the x, y, G1, G2, Levy(D), etc.
        if t ≤ (2/3)* T then
            if rand ≤ 0:5, then
                Step 1: Expanded exploration (X1)
                Update the current solution using Equation (17)
                if Fitness(X1(t+1)) < Fitness(X(t)) then
                    X(t) = (X1(t+1))
                    if Fitness(X1(t+1)) < Fitness(Xbest(t)) then
                        Xbest(t) = X1(t+1)
                    end if
                end if
            else
                Step 2: Narrowed exploration (X2)
                Update the current solution using Equation (19)
                if Fitness(X2(t+1)) < Fitness(X(t)) then
                    X(t) = (X2(t+1))
                    if Fitness(X2(t+1)) < Fitness(Xbest(t)) then
                        Xbest(t) = X2(t+1)
                    end if
                end if
            end if
        else
            if rand≤0:5 then
                Phase 3: Expanded exploitation (X3)
                Update the current solution using Equation (27)
                if Fitness((X3(t+1)) < Fitness(X(t)) then
                    X(t) = (X3(t+1))
                    if Fitness(X3(t+1)) < Fitness(Xbest(t)) then
                        Xbest(t) = X3(t+1)
                    end if
                end if
            else
                Phase 4: Narrowed exploitation (X4)
                Update the current solution using Equation (28)
                if Fitness((X4(t+1)) < Fitness(X(t)) then
                    X(t) = (X4(t+1))
                    if Fitness(X4(t+1)) < Fitness(Xbest(t)) then
                        Xbest(t) = X4(t+1)
                    end if
                end if
            end if
        end if
    end for
end while
return The optimal solution (Xbest)
```

This phase begins with the Aquila attacking its prey on land with its stochastic movements as they approach it. A method called "walk and grab" can capture prey. Equation (28) describes this action mathematically.

$$X_4(t+1) = QF \times X_{best}(t) - (G_1 \times X(t) \times rand) - G_2 \times Levy(D) + rand \times G_1 \quad (28)$$

Formulated as i $X_4(t+1)$ is the solution of the next iteration of t that runs and captures the prey. The search strategy is balanced by QF(t) at iteration (t), which indicates the quality function used to balance the search strategy. Calculating QF(t) is formulated in Equation (29). G_1 is the motion of the Aquila used to track prey. This value is created using Equation (30), and rand is a random value between 0 and 1. G_2 is a variable that drops from 2 to 0. The value of G_2 is created using Equation (31). The maximum number of iterations and the current iteration are written as t and T. Mathematically, Levy(D) is calculated based on Equation (20). D denotes the dimensional space, and Levy(D) is the distribution function of the levy flight. Algorithm 1 shows the pseudocode of Algorithm Aquila.

$$QF(t) = \frac{2 \times rand() - 1}{(1-T)^2} \quad (29)$$

$$G_1 = 2 \times rand() - 1 \quad (30)$$

$$G_2 = 2 \times \left(1 - \frac{t}{T}\right) \quad (31)$$

2.5 Experimental data and procedures

This research uses data from six products from a case study company in Indonesia. The data of this study is presented in Table 1, and the optimization parameters determined by the Aquila Algorithm are presented in Table 2. Matlab R2021a was used as the platform for the optimization process, which involved a total of 1000 iterations and a population of 1000. Comparisons were made between the proposed algorithm and the WCWOA, GA, PSO, Exact, and Heuristic methods.

3. Results and Discussion

3.1 Multi-item lot-sizing model optimization

Table 3 displays the optimization results of applying the Aquila algorithm to the multi-item lot sizing model. The decision variable on cycle order (T) is 0.9006. Meanwhile, the safety factor for products 1 (k_1) to 6 (k_6) is 2.99 respectively. The optimization results show that the ETC of the optimization results is Rp104,990,833.

3.2 Performance Test

Based on the comparison results between the proposed algorithm and WCWOA, GA, exact procedure, and heuristics presented in Fig. 2, it can be shown that the proposed algorithm has good performance. The findings show that the proposed algorithm produces the optimal ETC, Rp 104,990,833. The Exact WCWOA, PSO, and GA procedures produce comparable results to this algorithm. On the other hand, when compared to the heuristic procedure, the Aquila Algorithm can produce a cost savings of 0.84%. The Aquila algorithm has produced superior optimization results compared to other metaheuristic algorithms due to the findings presented here. This result is consistent with the findings of Sasmal, et al. [33] and Gao, et al. [36] in their respective studies. It has been shown that the Aquila algorithm has excellent performance and strong global exploration capabilities [37].

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Table 1. Research Data

Parameters	Value	
D_1	192	box
D_2	93	box
D_3	117	box
D_4	65	box
D_5	94	box
D_6	44	box
σ_1	16,4	box
σ_2	14,2	box
σ_3	23,4	box
σ_4	13,7	box
σ_5	19,9	box
σ_6	21,7	box
C_{is}	1234	Rp
H_1	417	Rp
H_2	250	Rp
H_3	167	Rp
H_4	417	Rp
H_5	250	Rp
H_6	167	Rp
P_1	168000	Rp
P_2	171000	Rp
P_3	175000	Rp
P_4	168000	Rp
P_5	171000	Rp
P_6	175000	Rp
L_1	504000	Rp
L_2	513000	Rp
L_3	525000	Rp
L_4	510000	Rp
L_5	525000	Rp
L_6	540000	Rp
B_1	0,5	Kg
B_2	0,7	Kg
B_3	0,8	Kg
B_4	0,7	Kg
B_5	0,9	Kg
B_6	1	Kg
j	8	Km
α	40000	Rp
β	6800	Rp
FL	3	Km/liter
V_{cap}	556	box

Table 2. Aquila Algorithm Parameters

Data	Value
Objective function	Minimize ETC
Number of population (search agent)	1000
Dimensions	7
Maximum iterations	1000
Upper bound (ub)	[1 2.99 2.99 2.99. 2.99 2.99 2.99]
Lower bound (lb)	[0 0 0 0 0 0 0]

Table 3. Optimization results with Aquila algorithm procedure

Decision variable	T	k_1	k_2	k_3	k_4	k_5	k_6
	0.9006	2.99	2.99	2.99	2.99	2.99	2.99
ETC	Rp104,990,833						

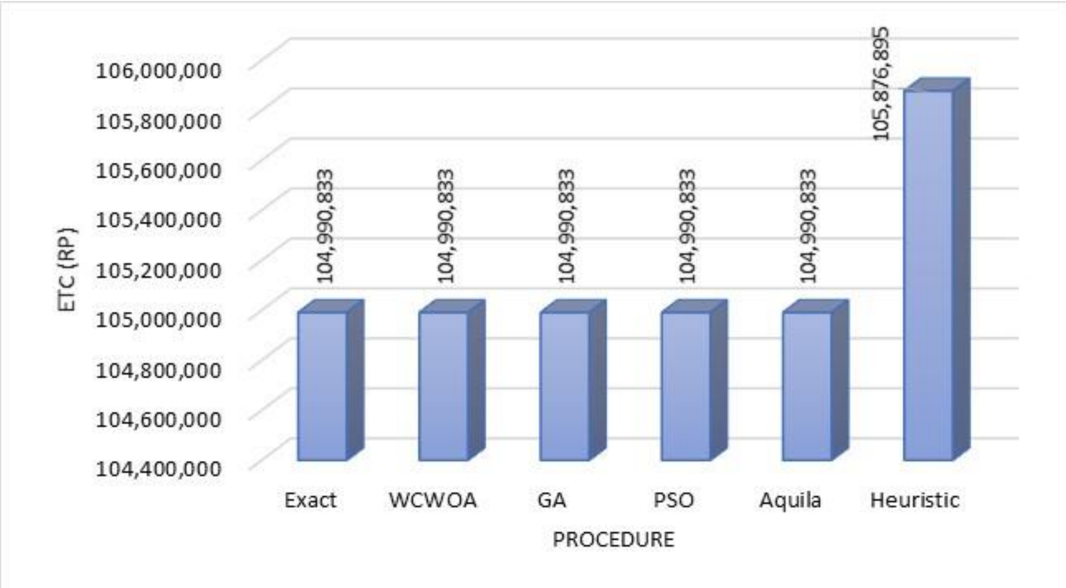

Fig. 2. ETC comparison

3.3 Sensitivity Analysis

3.3.1 Fuel Price Change (β)

Fig. 3 is the result of the sensitivity analysis of fuel price changes to the expected total inventory cost (ETC), fuel consumption, order quantity (Q), order cycle (T), and safety factor (k). The results show that an increase in fuel price can increase the expected total inventory cost (ETC), order quantity (Q), order cycle (T), and safety factor (k). However, an increase in fuel prices can reduce fuel consumption. Conversely, a decrease in fuel price can reduce order quantity (Q), order cycle (T), and safety factor (k). However, a decrease in fuel prices can increase fuel consumption.

The research findings state that an increase in fuel prices can increase order quantity (Q), order cycle (T), and safety factor (k). Increasing fuel prices can encourage companies to order more products (Q) at a time in response to higher transportation costs, resulting in larger inventories [38]. In addition, an increase in fuel prices may also lengthen the ordering cycle (T), as companies tend to order larger quantities to avoid frequent shipping costs [39]. Furthermore, an increase in fuel price can also increase the safety factor (k) in inventory, which can help overcome the uncertainty of fluctuating fuel prices [27]. However, on the other hand, an increase in fuel price may also reduce fuel consumption in transportation, as firms will be more cautious in using this resource. Therefore, these findings point to the complexity of managing inventory in an environment with fluctuating fuel prices, where firms must balance reducing transportation costs and minimizing the risk of excessive inventory [40].

3.3.2 Standard Deviation Change (σ)

Fig. 4 is the result of the sensitivity analysis of standard deviation to expected total inventory cost (ETC), fuel consumption, order quantity (Q), order cycle (T), and safety factor (k). An increase in the standard deviation of demand indicates an increase in uncertainty in product demand. The results show that increasing the standard deviation of demand can increase the expected total inventory cost (ETC), order quantity (Q), order cycle (T), and safety factor (k). However, increasing the standard deviation of demand can reduce fuel consumption. Conversely, a decrease in the standard deviation of demand can decrease the expected total inventory cost (ETC), order quantity (Q), order cycle (T), and safety factor (k). However, decreasing the standard deviation of demand can increase fuel consumption.

This study revealed that increasing the standard deviation of demand in the inventory model significantly impacts various key factors. An increase in demand standard deviation tends to increase the Expectation Total Cost (ETC) in the inventory system [41]. This happens because uncertainty in demand leads to an increased risk of inventory insufficiency, which forces companies to order more goods and, as a result, increases the overall inventory cost [42]. In addition, an increase in the standard deviation of demand also impacts the order quantity (Q), ordering cycle (T), and safety factor (k). Higher uncertainty requires larger orders (Q) and more frequent ordering cycles (T) to cope with demand fluctuations. In addition, the safety factor (k) also needs to be increased to protect the inventory from potential shortages. However, it should be noted that increasing the standard deviation of demand can have a positive impact, namely reducing fuel consumption [43]. This is because an increased demand standard deviation indicates increased product demand uncertainty. However, an increase in demand standard deviation can also decrease fuel consumption. This is due to increased safety stock, which reduces order frequency and fuel consumption [44].

3.3.3 Change in Vehicle Capacity

In Fig. 5 are the results of the sensitivity analysis of changes in vehicle capacity to the expected total cost of inventory (ETC), fuel consumption, order quantity (Q), order cycle (T), and safety factor (k). The results show that increasing vehicle capacity can reduce expected total inventory cost (ETC) and fuel consumption. However, increasing vehicle capacity can increase order quantity (Q), order cycle (T), and safety factor (k).

The findings of this study reveal that increasing vehicle capacity has a significant impact on the inventory model. On the one hand, increasing vehicle capacity can reduce expected total inventory cost (ETC) and fuel consumption. This can happen because a larger vehicle capacity enables the delivery of more goods on each delivery cycle, reducing the inventory costs associated with stock management [11]. In addition, a reduction in the number of trips required can also reduce fuel consumption. However, on the other hand, an increase in vehicle capacity can lead to an increase in order quantity (Q), order cycle (T), and safety factor (k). This could mean the company needs to order more goods in a delivery cycle and increase safety stock due to less order frequency [11].

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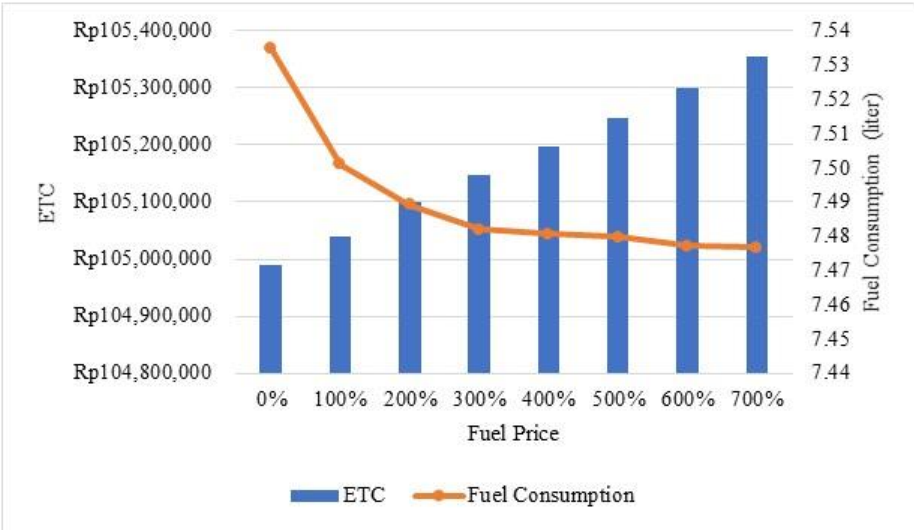

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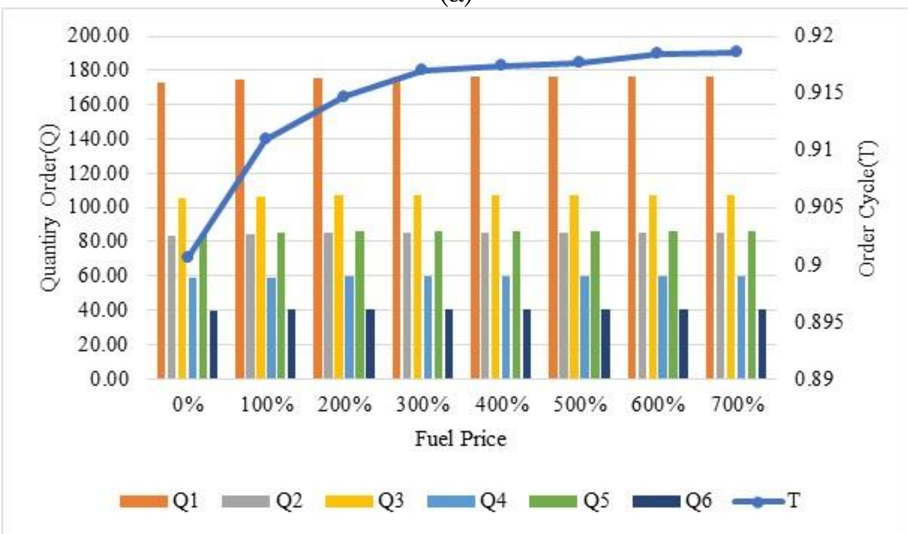

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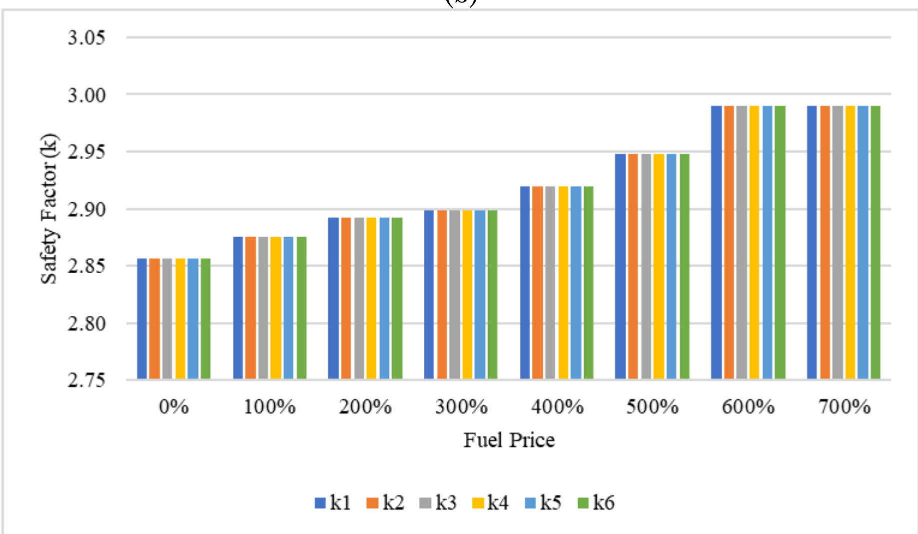

(c)

Fig. 3. Sensitivity analysis of fuel process changes on (a) ETC and fuel consumption; (b) T and Q; (c) safety factor (k)

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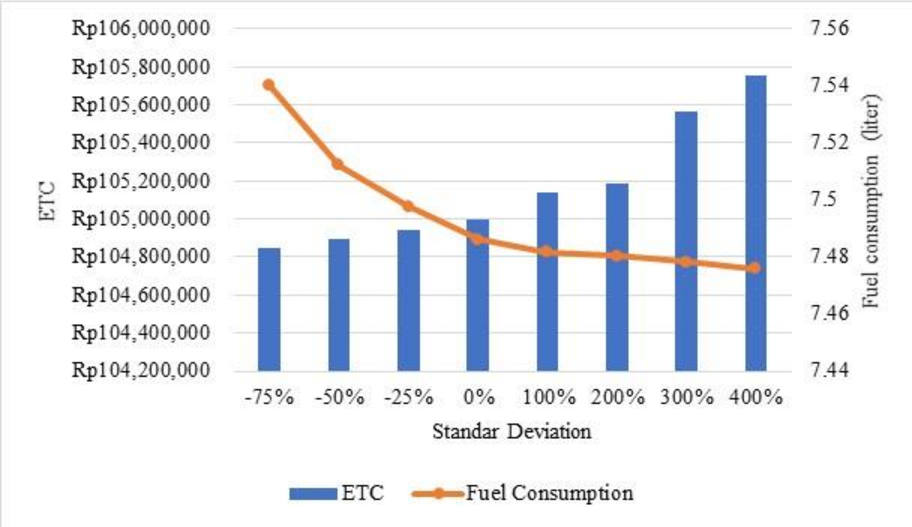

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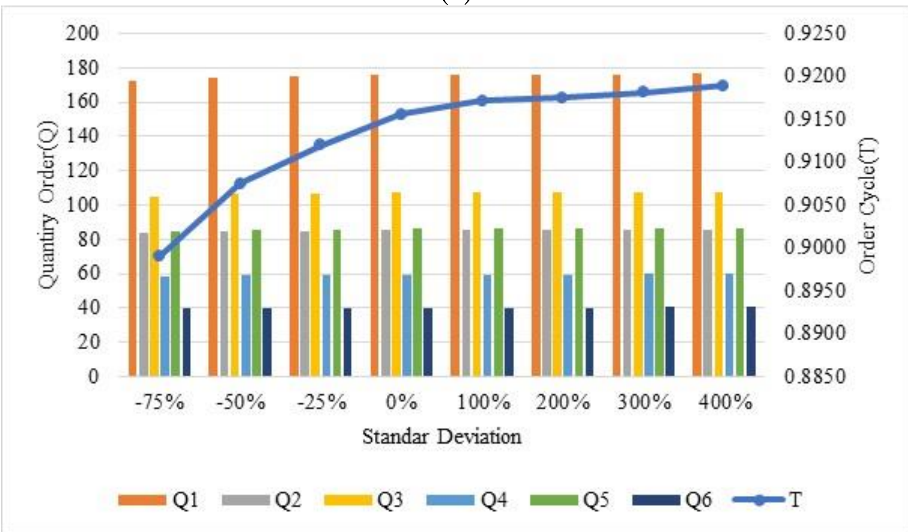

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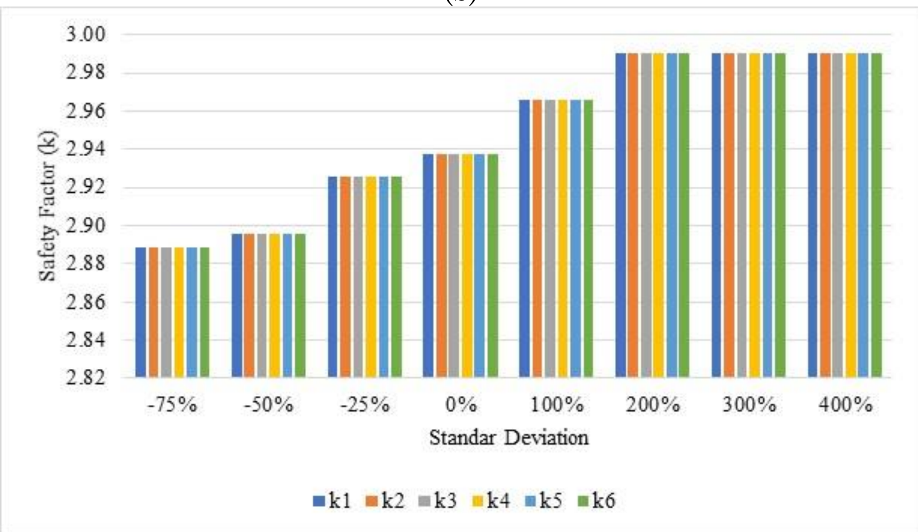

(c)

Fig. 4. Sensitivity analysis of standard deviation change on (a) ETC and fuel consumption; (b) T and Q; (c) k

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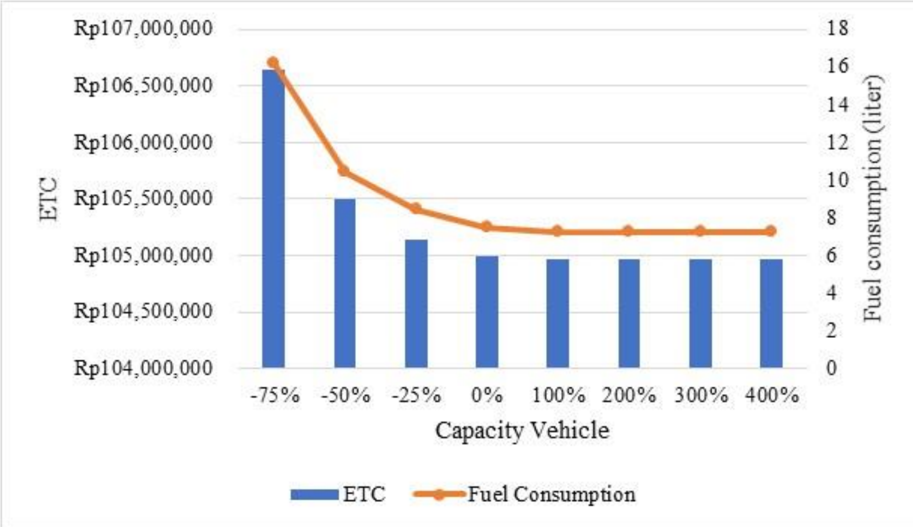

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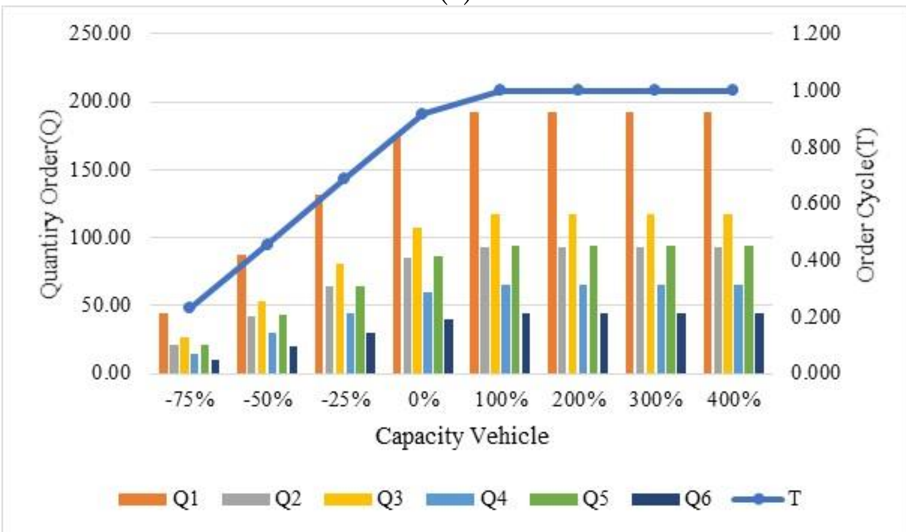

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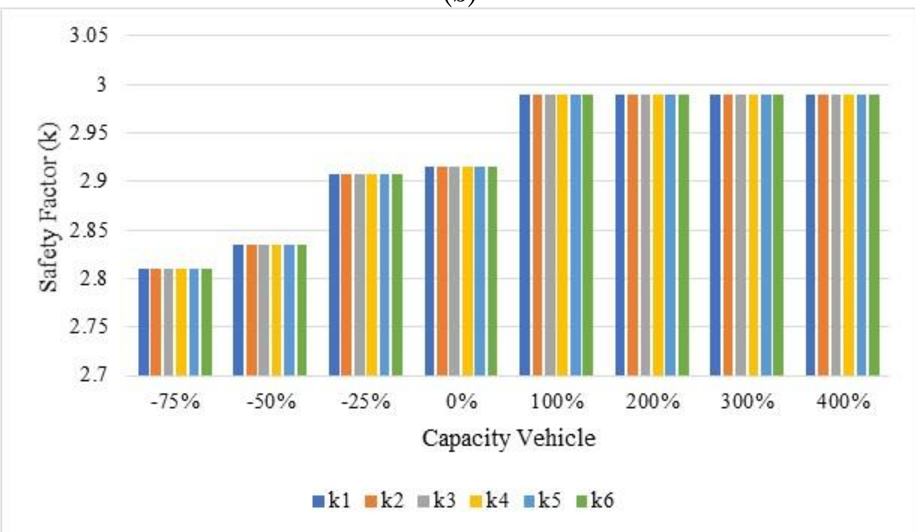

(c)

Fig. 5. Sensitivity analysis of vehicle capacity on (a) ETC and fuel consumption; (b) T and Q; (c) k

3.4 Implications

The findings of this research have significant implications for the development of understanding and practice regarding multi-item inventory problems involving transportation costs and capacity constraints in stochastic demand situations. This research not only expands the scope of more realistic inventory problems, but also presents an innovative solution in the form of a new procedure called "Aquila Algorithm" to optimize the model. The successful development of this algorithm opens the door for alternative optimization procedures in dealing with increasingly complex inventory problems in various industries. With the implementation of the Aquila Algorithm, companies and organizations can optimize their resource allocation, reduce transportation costs, and minimize the impact of uncertain demand fluctuations. This is an important step in improving operational efficiency and facing challenges in inventory management in an era of uncertainty.

The practical implication of the findings of this study is that companies should understand the impact of fuel price sensitivity on their inventory models. This study shows that an increase in fuel prices can negatively impact various aspects, including expected total inventory cost (ETC), order quantity (Q), order cycle (T), and safety factor (k). Therefore, companies should pay attention to fuel price fluctuations in their inventory planning and look for ways to mitigate the negative impact. While an increase in fuel prices may increase ETC, they should also consider potential savings in fuel consumption that could offset a significant portion of those costs. This may involve improving the efficiency of the vehicle fleet or seeking more efficient alternative energy sources. By understanding the implications of fuel price sensitivity, companies can proactively manage their inventory more efficiently and sustainably.

Based on the demand standard deviation sensitivity analysis, this research has important practical implications for companies' inventory management and logistics operations. While an increase in demand standard deviation may increase the expected total inventory cost (ETC), order quantity (Q), order cycle (T), and safety factor (k), it may also provide opportunities to reduce fuel consumption. In the face of this dilemma, companies must adopt a balanced approach to managing their inventory. They should carefully consider how much increase in the standard deviation of demand they can accept without significantly compromising their inventory costs. In addition, companies should also consider improving fuel efficiency in their logistics operations to compensate for the impact of increased demand standard deviation on fuel consumption.

Based on the sensitivity analysis of vehicle capacity, the findings of this study suggest that companies managing their supply chains need to carefully consider decisions related to increasing vehicle capacity. On the one hand, increasing vehicle capacity can significantly reduce inventory costs and fuel consumption, which can be a substantial driver of efficiency and savings. However, remember that this can also increase order quantity, order cycle, and safety factor, which can affect ordering costs and overall inventory management complexity.

4. Conclusion

In this research, a multi-item inventory model involving transportation costs and capacity constraints in stochastic demand situations has been successfully developed. In addition, this study also proposes a new procedure called the Aquila Algorithm to optimize the model, which can be applied as an alternative optimization procedure in complex inventory problems. The sensitivity analysis results show some important findings. An increase in fuel price can increase the expected total inventory cost (ETC), order quantity (Q), order cycle (T), and safety factor (k). Conversely, it can decrease fuel consumption. In addition, an increase in demand standard deviation can increase ETC, Q, T, and k but can also reduce fuel consumption. Increasing vehicle capacity can reduce ETC and fuel consumption but can increase Q, T, and k.

Based on this study's findings, several areas can be the focus of future research to better understand and improve inventory management in the context of multi-item problems with transportation costs and capacity constraints under stochastic demand. Further research can examine inventory management strategies that integrate aspects such as fuel price fluctuations, demand, and vehicle capacity to find optimal solutions for these various factors. In addition, further exploration of improvements or modifications of the Aquila Algorithm, or the development of other algorithms that are more efficient and effective in addressing this inventory problem, could be an interesting area of research. Future research could also consider other factors that affect inventory costs, such as sustainability aspects and the environmental impact of fuel consumption in the supply chain. Thus, future research can provide deeper insights into optimizing inventory management under increasingly complex and dynamic conditions.

Declarations

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