The Capacitated Sustainable EOQ Models: Models Considering Tax Emissions

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ABSTRACT

The study investigated problems of determining the lot size by considering sustainability and capital constraints for purchasing raw materials and taxes. Using the Sustainable EOQ (SEOQ) models that considered environmental aspects, the researchers also evaluated the capital constraints. The proposed models were used to minimize total inventory costs. In this study, there was a practical numerical analysis and sensitivity analysis to help decision-makers and inventory problems. Finally, the experimental results showed that the proposed models were effectively used to solve the problems.



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1. Introduction

Inventory is the primary key for a company to support its smooth production [1, 2]. Inventory has a vital role in the supply chain [3]. Besides, this has an impact on the company's operating costs [4, 5]. Several approaches have been proposed to solve inventory problems. These include Economic Order Quantity (EOQ), heuristic algorithms [6], and dynamic programming [7, 8]. Nowadays, environmental issues have become the concern of all companies and the world [9, 10]. Companies are required to pay attention to environmental aspects, including carbon emissions and waste [11]. Di In developed countries, the government provides a tax policy for carbon emissions to produce emissions and waste [12]. It is intended to raise company awareness of environmental problems [13, 14]. Recently, carbon emissions have become an essential issue in the industrial sectors. It includes transportation, inventory, and storage [15, 16]. Through inventory management, companies can control carbon emissions and economic aspects [17]. Researchers have currently shown their keen interest in Sustainable EOQ (SEOQ) [18].

In inventory, the SEOQ model is a lot size method used to determine economic orders by considering environmental aspects [19]. This model has to economically balance the financial perspective with the environmental perspective so that the industry can determine the appropriate policy that supports sustainability. There have been many studies about inventory issues that accommodate carbon emissions, such as Chen, et al. (2013), Jaber, et al. (2017), dan He, et al. (2015). In general, the studies consider issues of





carbon emission impacts, order frequency, and storage amount [23, 24]. Bauer, et al. (2010) proposed the SEOQ model by considering transportation costs. Subsequently, some researchers developed the model by adding tax cost on environmental impacts [26, 27]. Furthermore, some researchers also included environmental and tax costs in the procurement inventory model [28] and the production inventory model [29, 30]. Up to the present day, there have been several studies investigating the relationship between inventory [31] and capital constraints [32]. Raturi and Singhal (1990) developed the multi-item EOQ model by considering shortage cost, production, and capital constraints. Asadabadi (2016) also proposed the EOQ model with capital constraints for purchasing raw materials.

Although research on SEOQ has been flourishing lately, there have not been any studies that discuss SEOQ with capital restrictions for purchase and emission taxes. It has created a considerable amount of uncertainty about the relationship between SEOQ and capital constraints. Based on the description mentioned earlier, one way to overcome the problems is to accommodate capital limitation factors into the SEOQ models. The solution to the problems is based on the capital purchase function, which is the cost for the company's operations. Therefore, a new approach is needed to investigate the effect of capital in the SEOQ models. In the current study, we developed SEOQ models that considered carbon emissions by including capital restrictions on purchasing goods. This study aimed to develop a more sophisticated method to solve the problems of determining lot size by considering environmental impacts and capital constraints. Therefore, this research generated new insights in inventory, especially the SEOQ models with capital constraints. In addition, this study was intended to contribute to research on SEOQ by exposing the effects of capital on the number of lot sizes.

2. Methods

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2.1 Assumptions and Notations

: Demands

Assumptions of the SEOQ problems consist of (1) demands, order cost, purchasing cost, holding cost, tax fees, total emissions, and total capital in a fixed period; (2) components of the assumption that are deterministic; (3) each model is used for 1 item emission product; (4) tax cost per one emission unit; and (5) capital used is the capital for a purchase or emission tax. The notations used in the models include:

K: Cost per order : Purchasing cost per unit С h : Holding cost per unit f : Total emissions from ordering : Total emissions from purchasing v : Total emissions from holding g β : Lagrange multipliers М : Capital : Emission tax cost pQ: Number of orders Q_s : Optimal sustainable number of orders without tax Q_{sp} : Optimal sustainable number of orders with tax : Optimal sustainable number of orders without tax; with capital constrain Q_{sm} Q_{spm} : Optimal sustainable number of orders with tax and capital constrain TIC: Total inventory cost

 $TIC(Q_s)$: Optimum total inventory cost without tax

 $TIC(Q_{sp})$: Optimum total inventory cost with tax

 $TIC(Q_{sm})$: Optimum total inventory cost without tax; with capital constrains

 $TIC(Q_{spm})$: Optimum total inventory cost with tax and capital constraint

2.2. Proposed SEOQ Model without Limits

In terms of the proposed SEOQ model without limit, this study developed two SEOQ models, namely SEOQ model without tax and the SEOQ model with the tax on carbon emissions.

2.2.1 SEOQ without Tax

In this model, the researchers considered the costs included in the sustainable inventory, such as the fixed cost of an environmental impact for each cycle, order cost, purchasing, and holding cost. The total inventory cost is formulated as follows (1):

$$TIC = \frac{(K+f)\lambda}{Q} + \lambda(c+v) + \frac{(h+g)Q}{2}$$
(1)

To gain the optimal Q for the SEOQ model without tax, equation (1) is differentiated to Q. The result is presented in equation (2).

$$Q_s = \sqrt{\frac{2(K+f)\lambda}{(h+g)}} \tag{2}$$

Further, equation (2) is substituted to equation (1) to determine the SEOQ total inventory cost (TIC) without tax $(TIC(Q_s))$. $TIC(Q_s)$ is shown in equation (3).

$$TIC(Q_s) = \lambda(c+v) + \sqrt{2(K+f)\lambda(h+g)}$$
(3)

2.2.2 SEOQ with Tax

In the problem of SEOQ with emission tax, the study considered emission tax. The SEOQ model with tax is formulated in equation (4).

$$TIC = \frac{(K+pf)\lambda}{Q} + \lambda(c+pv) + \frac{(h+pg)Q}{2}$$
(4)

Equation (4) is differentiated to Q to produce the optimal Q for the SEOQ model with emission tax. The result is presented in equation (5).

$$Q_{sp} = \sqrt{\frac{2(K+pf)\lambda}{(h+pg)}} \tag{5}$$

Further, equation (2) is substituted to equation (1) to determine the total inventory cost (TIC) without tax $(TIC(Q_s))$. $TIC(Q_s)$ is shown in equation (3).

Further, equation (5) is substituted to equation (4) to determine the SEOQ total inventory cost (TIC) with emission tax $(TIC(Q_{sp}))$. $TIC(Q_{sp})$ is displayed in equation (6).

$$TIC(Q_{sp}) = \lambda(c+pv) + \sqrt{2(K+pf)\lambda(h+pg)}$$
(6)

2.3 Proposed SEOQ Model with Capital Constraint

The researchers developed another model concerning capital constraint. The capital referred to the cost used in purchasing raw materials and purchase emission tax. Equation (7) denotes the capital of raw material purchase without emission tax. Equation (8) represents the capital of raw material purchase without emission tax.

$$M = Q \times (c + v) \tag{7}$$

$$M = Q \times (c + pv) \tag{8}$$

The SEOQ model with capital constraints comprised two types: capital with tax and capital without tax, which has been developed by adding a constraint function, i.e., capital constraints.

2.3.1 Proposed SEOQ Model without Tax and with Capital Constraint

The study employed the Lagrange function to minimize the total inventory cost against the constraint. The model of total inventory cost in equation (1) is added to the constraint function in equation (7). The Lagrange function of the proposed SEOQ model without tax and with capital constraint is displayed in equation (9):

$$L(Q,\beta) = \frac{(K+f)\lambda}{Q} + \lambda(c+\nu) + \frac{(h+g)Q}{2} - \beta(Q(c+\nu) - M)$$
(9)

Further, equation (9) is differentiated partially to Q and β . The results are formulated in equations (10) and 11.

$$Q_{sm} = \sqrt{\frac{2(K+f)\lambda}{(2c+2\nu)\beta - g - h}} \tag{10}$$

$$\beta = \frac{-2(c+\nu)^2(K+f)\lambda + M^2(h+g)}{2M^2(c+\nu)}$$
(11)

Equations (10) and (11) are substituted to equation (4) to determine the total inventory cost (TIC) with capital constrain $(TIC(Q_{sm}))$. $TIC(Q_{sm})$ is presented in equation (12).

$$TIC(Q_{sm}) = \frac{(c+\nu)(\lambda\sqrt{(2c+2\nu)\beta-g-h}+\beta\sqrt{2\lambda(K+f)})}{\sqrt{(2c+2\nu)\beta-g-h}}$$
(12)

2.3.2 Sustainable SEOQ Model with Tax and Capital Constrain

In the sustainable SEOQ model with tax and capital constraint, the researchers developed the Lagrange formula from equation (4) by adding the constraining factor, i.e., equation (8). As a result, it is presented in equation (13).

$$L(Q,\beta) = \frac{(K+pf)\lambda}{Q} + \lambda(c+pv) + \frac{(h+pg)Q}{2} - \beta(Q(c+pv) - M)$$
(13)

Then, equation (13) is differentiated partially to Q and β . The result is formulated in equations (14) and (15). $TIC(Q_{spm})$ is displayed in equation (16).

$$Q_{spm} = \sqrt{\frac{2(K+pf)\lambda}{(2c+2pv)\beta - pg - h}}$$
(14)

$$\beta = \frac{-2(c+pv)^2(K+pf)\lambda + M^2(h+pg)}{2M^2(c+pv)}$$
(15)

$$TIC(Q_{spm}) = \frac{(c+pv)(\lambda\sqrt{(2c+2pv)\beta-pg-h}+\beta\sqrt{2\lambda(K+pf)})}{\sqrt{(2c+2pv)\beta-pg-h}}$$
(16)

2.4 Numerical Experiment Procedure

This section shows the numerical experiment procedure on the proposed SEOQ models. The experiment was carried out to test the sensitivity of the proposed models. The data are presented in Table 1.

Table 1. Experiment Data					
Variable	Unit	Value			
λ	Kg	50			
K	\$/order	40			
c	\$/kg	12			
p	\$/kg	2			
h	\$/kg	2			
f	$kgCO_2$	60			
υ	$kgCO_2$	5			
g	$KgCO_2$	1			

In the numerical experiment, this study used 20 variants of capital purchase data and emissions tax. The experiment was also conducted to find out the effects of Q lot size on the inventory cost.

3. Results and Discussion

3.1. Numerical Analysis

Table 2 and Table 3 contain the results of the experiments using the four proposed models. Table 2 recapitulates the results of the SEOQ model without tax. The findings indicate that ordering lots of SEOQ models without capital constraint reaches 58 units for each order annually. The total inventory cost is as much as \$ 1023,2. According to the SEOQ experiment without tax in Table 2, if the available capital is relatively small, the lot size of orders is comparatively small. On the other hand, if the capital is relatively big, the lot size of orders is also relatively big. As reflected in Fig. 3, if $\beta > 0$, Q is optimal by using equation (2). Conversely, if $\beta < 0$, equation (12) is employed.

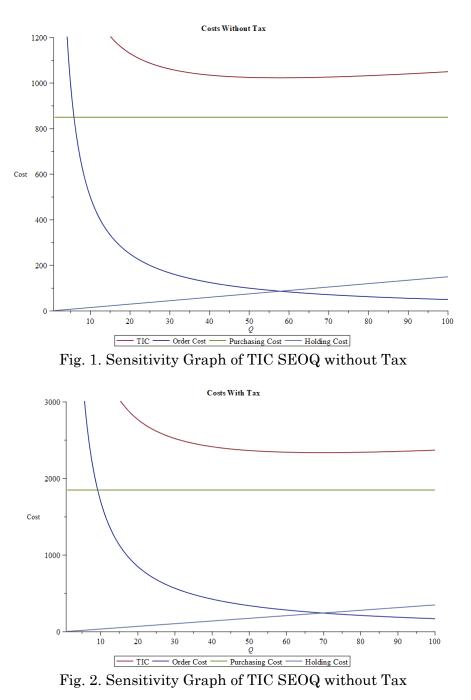
Table 3 summarizes the results of the SEOQ model with tax. The findings show that the number of ordering lot of SEOQ model without capital constraints obtains 64 units per order annually. The total inventory cost is \$ 1353. According to the SEOQ experiment without tax in Table 3, if the available capital is relatively small, the lot size of orders is relatively small. However, if the capital is relatively big, the lot size is also relatively big. According to Fig. 3, if $\beta > 0$ is optimal by using equation (5). Nonetheless, if $\beta < 0$, equation (15) is used.

SEOQ without tax							
Without o	apital constraint	With capital constraint					
Q_s	$TIC(Q_s)$	M	β	Q_{sm}	$TIC(Q_{sm})$		
		50	-33.910	3	2555.1		
		150	-3.680	9	1430.1		
		250	-1.270	15	1212.2		
		350	-0.600	21	1123.8		
		450	-0.330	27	1078.7		
		550	-0.190	32	1053.1		
		650	-0.110	39	1038.1		
		750	-0.060	45	1029.5		
		850	-0.020	53	1024.9		
58	1023.2	950	-0.010	55	1023.3		
90	1023.2	1050	0.010	62	1023.6		
		1150	0.020	66	1025.3		
		1250	0.030	72	1028.2		
		1350	0.041	79	1032.1		
		1450	0.047	85	1036.6		
		1550	0.052	91	1041.6		
		1650	0.057	98	1047.1		
		1750	0.060	103	1053.0		
		1850	0.063	108	1059.2		
		1950	0.065	113	1065.6		

Table 2. Experiment Results of SEOQ without Tax

Table 3. Experiment Results of SEOQ with Tax SEOQ without tax

Without capital constraint		With capital constraint			
Q_{sp}	$TIC(Q_{sp})$	M	β	Q_{spm}	$TIC(Q_{spm})$
		50	-70.309	3	4897.6
		150	-7.731	7	2287.8
		250	-2.725	12	1828.8
		350	-1.345	16	1634.7
		450	-0.778	21	1532.3
		550	-0.490	26	1470.5
		650	-0.325	30	1429.9
		750	-0.221	35	1403.0
		850	-0.152	39	1384.3
64	1353	950	-0.104	44	1371.7
04	1999	1050	-0.068	48	1363.1
		1150	-0.042	53	1357.6
		$1250 \\ 1350$	-0.021	58	1354.5
			-0.005	62	1353.1
		1450	0.007	66	1353.2
		1550	0.017	71	1353.5
		1650	0.026	75	1356.7
		1750	0.033	80	1359.7
		1850	0.039	84	1363.3
		1950	0.044	89	1367.5



3.2. SEOQ Sensitivity Analysis with Capital Constraint

Fig. 1 and Fig. 2 present the effects of lot size (Q) on the inventory cost. The results imply that the order cost and total inventory cost of the SEOQ model are affected by Q. For order cost, the greater the value of Q, the smaller the cost. It is due to the number of shipping frequencies that gets smaller. Conversely, the smaller the Q value, the higher the order cost. In terms of inventory cost, the higher the Q value, the greater the total inventory cost. Conversely, the smaller the value of Q, the smaller the total inventory cost.

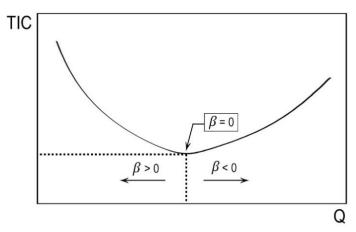


Fig. 3. Correlation between β and TIC

4. Conclusion

The purpose of this study was to develop a more sophisticated method for solving the problems of determining lot size by considering environmental impact and capital constraint. In this study, the researchers developed SEOQ models by considering capital constraints. Capital itself is the cost used in the process of purchasing raw materials. Capital is an essential aspect that needs to be considered in decision making. This research also presented numerical experiments and analyzed the sensitivity of the models. The experimental results show that the proposed models helped solve the problems. Further, it is suggested that future researchers develop SEOQ models for multi-item products with capital and capacity constraints.

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