

# A Routing Model for Hybrid Hub-and-Spoke with Time Windows

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## ABSTRACT

Currently, the company competition is getting tighter. Distribution companies need to provide excellent service to their customers to maintain their competitiveness. Distribution service performance could be measured with lead time. However, Reducing lead times may increase costs. This problem could be solved using freight consolidation and reducing backhaul. Freight consolidation can be done by using a hub-and-spoke network with a combination of inbound and outbound distribution. This study developed a route model for hybrid hub-and-spoke with time windows. This model determined the routes for shipping goods to consumers and taking products to suppliers using the same vehicle to reduce the backhaul. This model also conducted freight consolidation at the hub. The decision variables in this model included the routes of delivery to consumers, the collection of goods at the suppliers, the number of products distributed through the hub and direct shipping, and the excellent distribution route. This model was implemented into the problems. Besides, a sensitivity analysis of the model was carried out.



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## 1. Introduction

Currently, competition among companies is getting tighter. In order to be competitive, distribution companies need to provide excellent service to customers. The distribution service performance can be measured with lead time. However, the reduction of lead time can result in the underutilization of a vehicle [1]. It may increase transportation costs significantly. Moreover, distribution companies in Indonesia deal with inbound and outbound distribution with separate fleets. They use a particular fleet to deal with outbound distribution, and another fleet deals with inbound distribution. It causes a backhaul as the fleet used to pick up goods at the suppliers is empty when they depart. Meanwhile, the fleet for delivering the products to the customers is also vacant as



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going back to the depot. The reduction of lead time may cause an increase in transportation costs. Therefore, it is necessary to develop a strategy to reduce backhaul, increase vehicle utilization, and reduce lead time [1]. Combining routing strategy inbound and outbound with hub-and-spoke consolidation may solve the problems.

Research on the route was firstly introduced by Dantzig and Ramser in 1959 [2]. The problem is called the Vehicle Routing Problem (VRP). Since then, studies on variations of VRP have emerged. There have been some variations of models developed to solve VRP. Imran and Okdinawati [3], Toth and Vigo [4], Shi, et.al. [5] have developed a heuristic approach to solve VRP. There have also been some studies aiming at solving VRP with time windows (VRPTW), such as El-Sherbeny [6], Liu and Jiang [7], and Ayu and Putri [8]. These variations consider the time windows of customers in determining the routes. Li and Liu [9], Goetschalckx and Jacobs-Blecha [10], Saremi [11] developed another variation of VRP called Vehicle Routing Problem with Backhaul (VRPB). They determine routes to deliver goods to customers and pick up goods at the supplier's using the same fleet. The combination of the abovementioned variations of VRP, also called VRP with backhaul and time windows, has been investigated by Koch, et al. [12], Gupta [13].

There have been various studies on hub-and-spoke. Enis and Mehmet [14] proposed a new multi-objective mathematical model for the newly constructed hub and spoke system. Zhalecian, et al. [15] suggested a new framework to design a hub network under the operational disruption risks. Alkaabneh, et al. [16] proposed a model for hybrid hub-and-spoke network design with inter-hub economic scale and hub congestion. Lee and Moon [17] solved a problem about a hub-and-spoke scheme in Korean postal logistics with realistic restrictions; they determined the location of the hub and flow of the goods. Chen, et al. [18] formulated a model to optimize the operational cost of a hub-and-spoke network operation for less than the truckload transportation service; the model also incorporated the incremental quantity discount. Yan, et al. [19] formulated a model that determines the transportation volume between distribution groups. There have been some studies that focus on determining routes for hybrid the hub-and-spoke scheme. Liu, et al. [20] formulated a model that determines the route that allows both hub-and-spoke and direct delivery shipment modes. Jian, et al. [21] developed a model that determines the transshipment route of similar products with various vehicle capacities in the hub-and-spoke scheme. Wasner and Zäpfel [22] formulated a model that determines the route of pickup, delivery, and transshipment in the hub-and-spoke scheme.

As mentioned earlier, many kinds of research have studied VRP and the hub-and-spoke scheme. However, none of them develops a model that can determine a route to deliver and pick up goods using the same fleet and route for distribution. Moreover, there have not been any studies using the hub-and-spoke scheme considering time windows at customers and suppliers (time restriction at the hub that can change depending on the number of products). This research aims at developing a model to determine the route for hybrid hub-and-spoke with time windows. There are several important areas where this study makes its original contribution to the new routing model for the hybrid hub-and-spoke with time windows. This research covers five sections, including introduction, model formulation, implementation and sensitivity analysis, result and discussion, and conclusion. The introduction describes the background of problems and state-of-the-art of the study. The formulation of the model is to be discussed in section two. The model is presented in the implementation section and then followed with the sensitivity analysis. The results and discussion section discusses the findings of the implementation analysis and model sensitivity. Finally, the results will be explained in conclusion.

## 2. Methods

### 2.1 Assumptions, Notations, and Model Formulation

The assumptions of the model comprise of 1). The constructed model only uses one hub as a consolidation center; 2). The model's performance is measured with the transportation expenses that incorporate only gas and vehicle usage costs. 3). The loading and unloading times are ignored; 4). The demand is deterministic, and 5). The sorting time/ unit is deterministic. The notations used in this model include:

$D$	: set of DC ( $D = 1, 2, \dots, dn$ )
$J$	: set of suppliers and customers
$J_d$	: set of all nodes
$J_k$	: set of all customers ( $k = d_{n+1}, d_{n+2}, \dots, k_n + d_{n+1}$ )
$J_s$	: set of all suppliers ( $s = k_{n+1}, k_{n+2}, \dots, S_n + k_{n+1}$ )
$J_{DK}$	: set of all customers and DC
$J_{Ds}$	: set of all suppliers and DC
$V$	: set of delivering fleets ( $1, 2, \dots, v$ )
$a_d$	: earliest fleet departure time on DC $d$ , $d \in D$
$a_j$	: earliest service time of customers and suppliers, $j \in J$
$b_j$	: latest service time of customers and suppliers, $j \in J$
$C_{ij}$	: distance between nodes $i$ and $j$
$d_j$	: the demand of $j$ customers, $j \in J_K$
$g_v$	: vehicle usage cost
$GC_1$	: gas cost/km small vehicle
$GC_2$	: gas cost/km big vehicle
$M$	: <i>big constant</i>
$P_i$	: supplier demand $i \in J_s$
$Q_v$	: the capacity of vehicle $v$
$S_j$	: service time of customers or suppliers $j$ , $j \in J$
$t_{ij}$	: traveling time between nodes $i$ and $j$
$L_v$	: initial load of vehicle $v$ , $v \in V$ (load of the vehicle leaving the DC)
$S_v$	: the final load of the vehicle, $v \in V$ (load of the vehicle arriving at DC)
$T_d$	: departure time from DC
$T_j$	: time to begin service at customers/ suppliers, $j \in J$
$X_{ijv}$	: $\begin{cases} 1, \text{if vehicle } v \text{ travels directly from node } i \text{ to } j \\ 0, \text{otherwise} \end{cases}$
$T_{md}$	: arrival time of the last vehicle at DC $d$
$T_{hd}$	: the earliest departure time of vehicle from DC $d$ to distribute goods to another depot
$T_{max}$	: latest arrival time allowed at the hub with the assumption that all goods are distributed via the hub
$W_{dv}$	: arrival time of vehicle $v$ at DC $d$
$Tl$	: latest arrival time at DC
$P_t$	: sorting time/unit
$M_{dd'}$	: quantity moved from DC $d$ to DC $d'$
$T_h$	: latest arrival time at the hub
$M_{dh}$	: quantity moved from DC $d$ to hub $h$
$M_{hd}$	: quantity moved from hub $h$ to DC $d$
$X_{ddv}^1$	: $\begin{cases} 1, \text{if small vehicle } v \text{ travels directly from DC } d \text{ to } d' \\ 0, \text{otherwise} \end{cases}$

- $X_{ddrv}^2$  : {1, if big vehicle v travels directly from DC d to d'}  
 { 0, otherwise }  
 $D_h$  : set of DC and hub (1,2, ...,  $d_{n+1}$ )  
 $H$  : index of hub ( $d_{n+1}$ )  
 $n$  : quantity of vehicle for distribution at each DC  
 $V_n$  : the sum of all vehicle for distribution purpose (1,2, ...,  $n \times d$ )  
 $V_{n1}$  : distribution vehicle DC 1 (1,2, ...,  $n_1$ )  
 $V_{n2}$  : distribution vehicle DC 2 ( $n_{1+1}, n_{1+2}, \dots n_2$ )  
 $V_{n3}$  : distribution vehicle DC 3 ( $n_{2+1}, n_{2+2}, \dots n_3$ )  
 $K^1$  : small vehicle capacity  
 $K^2$  : big vehicle capacity

The model is developed based on Saremi [11] and Liu, et al. [20]. This model has four decision variables. Three decision variables are to determine the route of a vehicle, such as  $X_{ijv}$ ,  $X_{ddrv}^1$ , and  $X_{ddrv}^2$ .  $X_{ijv}$  is a decision that arranges the route of shipping the goods to customers and pick up the goods.  $X_{ddrv}^1$  refers to a decision to arrange small vehicle routes for transshipment in the hub-and-spoke network.  $X_{ddrv}^2$  is a decision that organizes the big vehicle route for transshipment in the hub-and-spoke network. One decision variable determines whether the distribution is carried via a hub or direct shipping along with the quantity. The objective function of this model is to minimize the total costs, including vehicle usage cost (VC) and transportation cost (TC). The following is the route determination model for the hybrid hub-and-spoke distribution network.

### Objective Function

Minimize  $VC + TC$

Where:

$$VC = \sum_{d \in D} \sum_{v \in V} \sum_{j \in J} g_1 X_{djv} + \sum_{d \in D} \sum_{v \in V_n} \sum_{d' \in Dh, d' \neq d} g_1 X_{dd',v}^1 V_{dv} + \sum_{d \in D} \sum_{v \in V_n} \sum_{d' \in Dh, d' \neq d} g_2 X_{dd',v}^2 V_{dv} \quad (1)$$

$$TC = \sum_{i \in J_d} \sum_{j \in J} \sum_{v \in V} X_{ijv} C_{ij} GC_1 + \sum_{d \in Dh} \sum_{d' \in dh, d' \neq d} \sum_{v \in V} X_{dd',v}^1 CC_{ij} GC_1 + \sum_{d \in Dh} \sum_{d' \in dh, d' \neq d} \sum_{v \in V} X_{dd',v}^2 CC_{ij} GC_2 \quad (2)$$

### Subject to

$$\sum_{i \in J_d} \sum_{v \in V} X_{ijv} = 1 \quad \forall j \in J \quad (3)$$

$$\sum_{i \in J_d} X_{ijv} = \sum_{i \in J_d} X_{jiv} \quad \forall j \in J, \forall v \in V \quad (4)$$

$$\sum_{j \in J} X_{djv} = \sum_{j \in J} X_{jdv} \quad \forall v \in V, \forall d \in D \quad (5)$$

$$\sum_{i \in D} \sum_{j \in J} X_{jiv} = 1 \quad \forall v \in V \quad (6)$$

$$Q_v \geq \sum_{i \in J_d} \sum_{j \in J_k} d_j X_{ijv} \quad \forall v \in V \quad (7)$$

$$Q_v \geq \sum_{i \in J_s} \sum_{j \in J_d} p_i X_{ijv} \quad \forall v \in V \quad (8)$$

$$T_j \geq T_{dv} + t_{dj} - M(1 - X_{djv}) \quad \forall j \in J, \forall v \in V, \forall d \in D \quad (9)$$

$$T_j \geq T_i + S_i + t_{ij} - M(1 - \sum_{v \in V} X_{ijv}) \quad \forall i \in J, \forall j \in J \quad (10)$$

$$a_d \leq T_{dv} \quad \forall v \in V, \forall d \in D \quad (11)$$

$$a_j \leq T_j \leq b_j \quad \forall j \in J \quad (12)$$

$$\sum_{j \in J_k} \sum_{v \in V} X_{ijv} = 0 \quad \forall i \in J_s \quad (13)$$

$$\sum_{i \in D} \sum_{j \in D, j \neq i} X_{ijv} = 0 \quad \forall v \in V \quad (14)$$

$$\sum_{i \in S} \sum_{j \in S, j \neq i} X_{ijv} \leq |s| - 1 \quad \forall s \in J, |s| \geq 2, \forall v \in V \quad (15)$$

$$W_{dv} = \sum_{i \in J_k} (T_i + S_i + t_{id}) \times X_{idv} \quad \forall v \in V \tag{16}$$

$$Tm_d = \text{Max} (W_{d1}, \dots, W_{dv}) \quad \forall d \in D \tag{17}$$

$$Tn_d = Tm_d + (Pt \times \sum_{d' \in D, d' \neq d} m_{dd'}) \quad \forall d \in D \tag{18}$$

$$Th = T_{\text{max}} + (Pt \times \sum_{d \in D} \sum_{d' \in D, d' \neq d} m_{dd'}) \tag{19}$$

$$m_{dh} + \sum_{d' \in D, d' \neq d} m_{dd'} = \sum_{d' \in D, d' \neq d} M_{dd'} \quad \forall d \in D \tag{20}$$

$$m_{hd} + \sum_{d' \in D, d' \neq d} m_{d'd} = \sum_{d' \in D, d' \neq d} M_{d'd} \quad \forall d \in D \tag{21}$$

$$m_{dd'} \leq \sum_{v \in V} (X_{dd'v}^2 + X_{d'dv}^2) * K^2 + \sum_{v \in V} (X_{dd'v}^1 + X_{d'dv}^1) * K^1 \quad \forall d \in D, \forall d' \in D, d' \neq d \tag{22}$$

$$m_{dh} \leq \sum_{v \in V} X_{dhv}^2 * K^2 + \sum_{v \in V} X_{dhv}^1 * K^1 \quad \forall d \in D \tag{23}$$

$$\sum_{v \in V} X_{dd'v}^2 = \sum_{v \in V} X_{d'dv}^2, \sum_{v \in V} X_{dd'v}^1 = \sum_{v \in V} X_{d'dv}^1 \quad \forall d \in D, \forall d' \in D, d' \neq d \tag{24}$$

$$\sum_{d' \in Dh} X_{dd'v}^1 = \sum_{d' \in Dh} X_{d'dv}^1 \quad \forall d \in d, \forall v \in V \tag{25}$$

$$\sum_{d' \in Dh} X_{dd'v}^2 = \sum_{d' \in Dh} X_{d'dv}^2 \quad \forall d \in d, \forall v \in V \tag{26}$$

$$\sum_{d' \in Dh} X_{dd'v}^1 \leq 1 \quad \forall d \in D, \forall v \in V \tag{27}$$

$$\sum_{d' \in Dh} X_{dd'v}^2 \leq 1 \quad \forall d \in D, \forall v \in V \tag{28}$$

$$V_{dv} = \begin{cases} 1, & \text{if vehicle } v \text{ came from } d \\ 0, & \text{if not} \end{cases}$$

$$Twh_{dv}^1 \geq \left( (1 - V_{dv}) \sum_{i \in D, i \neq d} (Tn_d + TT_{id}) \times X_{idv}^1 \right) + (V_{dv} (\sum_{i \in D, i \neq d} Twh_{id}^1 + TT_{id} + (Pt \times m_{id})) \times X_{idv}^1) \quad \forall d \in Dh, \forall v \in V \tag{29}$$

$$Twh_{dv}^2 \geq \left( (1 - V_{dv}) \sum_{i \in D, i \neq d} (Tn_d + TT_{id}) \times X_{idv}^2 \right) + (V_{dv} (\sum_{i \in D, i \neq d} Twh_{id}^2 + TT_{id} + (Pt \times m_{id})) \times X_{idv}^2) \quad \forall d \in Dh, \forall v \in V \tag{30}$$

$$Twh_{dv}^1 \leq Tl \quad \forall d \in D, \forall v \in V \tag{31}$$

$$Twh_{dv}^2 \leq Tl \quad \forall d \in D, \forall v \in V \tag{32}$$

$$Twh_{hv}^1 \leq Th \quad \forall h \in H, \forall v \in V \tag{33}$$

$$Twh_{dv}^2 \leq Th \quad \forall d \in H, \forall v \in V \tag{34}$$

The Objective function of this problem is to minimize the total distribution cost, including vehicle usage cost in equation (1) and transportation cost in equation (2). Constraint (3) ensures that each customer and supplier is served only one time. Constraint (4) ensures that every vehicle comes to and leaves the customer afterward. Constraint (5) confirms that each vehicle returns to its respective DC. Constraint (6) agrees that each vehicle departs from DC. Constraint (7) ensures that every vehicle's initial load does not exceed the capacity. Constraint (8) ensures that every vehicle's last load does not exceed the capacity.

Constraints (9)-(12) approve that every route does not violate the time windows. Constraint (9) calculates the arrival time at a customer or supplier's place if the vehicle departs from the DC. Constraint (10) calculates the arrival time at the customer's or supplier's place if the vehicle departs from another customer or supplier. Constant M in

constraints (9) and (10) prevents the unselected route that violates the time windows. If a specific route is not selected, the value of  $X_{d'jv}$  or  $X_{ijv}$  is 0. Hence, M will become a considerable deduction, and the constraint will not be violated. Constraint (11) guarantees that every vehicle does not depart before the earliest departure schedule. Constraint (12) ensures that every vehicle arrives at the customer's and supplier's place at the range of the time windows. Constraints (13)-(15) are the additional constraints from the previous model. Constraint (13) ensures that picking up the goods from the suppliers is to be carried out after the vehicle finishes the shipment to the customer. Constraint (14) prevents the vehicle from stopping at another depot while shipping the goods to the customer. Constraint (15) is a sub-tour elimination constraint.

Constraints (16)-(19) are the model's novelty to ensure that the proposed lead time is achieved. Constraint (16) calculates the arrival of each vehicle at DC. Constraint (17) determines the latest arrival of the vehicle at DC. Constraint (18) estimates the earliest departure time of each vehicle at every DC, using the  $T_{md}$  plus the sorting time at the depot. Constraint (19) calculates the latest arrival time of the vehicle at the hub. The latest arrival time is calculated by adding the  $T_{max}$  with the processing time of the shipped goods via direct shipping. If there is a direct delivery between DCs, the number of goods that need to be processed by the hub decreases. Therefore, the time limit of the hub becomes shorter. If the  $T_{max}$  is 23.00 and 60 goods shipped via direct shipping with sorting time of each good is one minute, this is 24.00 (23.00 plus sixty minutes). Constraints (20) and (21) guarantee that the goods moved from DC  $d$  to  $d'$  equal to the demand needed to be moved from DC  $d$  to  $d'$ . Constraint (22) is the capacity constraint for immediate shipping. In direct shipping, the vehicle that departs to another DC will come back with the goods from the destination to its respective DC. Also, the capacity of the vehicle needs to be considered for both directions. Constraint (23) is the capacity constraint for shipment via a hub. Constraint (24) ensures a different vehicle requirement in the shipment for both directions; it becomes the essential requirement for directions.

Constraints (25) and (26) ensure that all small and big vehicles return to their respective DC. Constraints (27) and (28) ensure that all small and big vehicles only leave DC once. Constraints (29) and (30) calculate the arrival time of each vehicle at DC.  $V_{dv}$  is an auxiliary variable that helps to identify the initial DC of each vehicle. Suppose the vehicle departs from the initial DC. In that case, the arrival time will be calculated using the earliest departure time from the DC plus the traveling time. Otherwise, the arrival time will be calculated using the vehicle's arrival time at the DC plus service and traveling time. Constraints (31) and (32) guarantee that every vehicle arrives at DC before the latest arrival time allowed. Constraints (33) and (34) ensure that each vehicle arrives at the hub before the latest arrival time that is permitted at the hub.

## 2.2 Data and Experimental Procedures

The model implemented in this study consisted of three DCs, eight customers, and four suppliers. It was inspired by a real case on the automotive wholesale company in Indonesia as it owns four different suppliers and three DCs. Table 1 describes the distance between hub, depot, customers, and suppliers. Nodes 1-3 were the DCs, nodes 4-11 were the customers, and 12-15 were the suppliers. With the velocity assumption of 40 km/hour, the traveling time between the nodes was calculated. Each customer had different demand, as described in Table 2.

Each supplier had different quantities of goods that need to be taken, as described in Table 3. The demands are shown in Table 2 and Table 3 (unit measurement in box).



Each customer and supplier had a specific time window. Time windows described the earliest and the latest arrival time of each customer and supplier. The vehicle was not allowed to arrive after the arrival service time; but, it was allowed to come before the earliest arrival time and wait until the initial arrival time. Table 4 was time windows and service time for each supplier and customer. The departure time for each vehicle from each DC was 8 AM. There was a distance between hub and depot.

Table 5 illustrates the gap between hub and depot. It shows that nodes 1-3 were DCs, and node 4 was the hub. The unit measurement employed in this table was kilometers. There was a quantity of transit between the DCs. Table 6 describes the quantity of transit between the DCs. There was a time limit on each DC and hub. The vehicle was not allowed to arrive at DC and hub after the time limit.

Table 7 shows the time limit on each DC and hub. The gas cost for the small vehicle was IDR 980 / km and IDR 1,250 for the big one. Usage cost for the small vehicles was as much as IDR 150,000 and IDR 225,000 for big vehicles.

Table 1. Distance Data

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	160	175	60	75	55	273	265	250	270	292	95	90	300	280
2	200	0	150	260	275	255	73	65	50	270	292	295	290	100	280
3	200	230	0	260	275	255	273	265	250	70	92	295	290	300	80
4	60	260	260	0	45	40	195	200	190	245	265	60	55	255	320
5	75	275	275	45	0	30	210	220	225	230	255	45	65	265	280
6	55	255	255	40	30	0	180	190	200	270	280	50	40	280	290
7	273	73	273	195	210	180	0	60	40	200	190	250	230	60	280
8	265	65	265	200	220	190	60	0	35	210	195	230	220	50	270
9	250	50	250	190	225	200	40	35	0	200	180	220	200	45	300
10	270	270	70	245	230	270	200	210	200	0	30	200	210	230	50
11	292	292	92	265	255	280	190	195	180	30	0	220	200	200	65
12	95	295	295	60	45	50	250	230	220	200	220	0	40	240	260
13	90	290	290	55	65	40	230	220	200	210	200	40	0	230	200
14	300	100	300	255	265	280	60	50	45	230	200	240	230	0	210
15	280	280	80	320	280	290	280	270	300	50	65	260	200	210	0

Table 2. Customer Demand

Customer	Customer demand
4	30
5	45
6	40
7	50
8	35
9	50
10	43
11	21

Table 3. Supplier Demand

Supplier	Supplier Demand
12	70
13	75
14	65
15	68

Table 4. Time Windows and Service Time

Customer dan Supplier	Time Windows	Service time
4	9-12	1
5	12-15	1
6	13-16	1
7	8-13	1
8	10-15	1
9	12-16	1
10	9-14	2
11	11-14	1
12	15-18	2
13	13-16	2
14	13-16	2
15	13-15	2

Table 5. Distance Between Hub and DCs

	1	2	3	4
1	0	160	175	80
2	160	0	150	87,5
3	175	150	0	90
4	90	87,5	90	0

Table 6. In transit Quantity

	1	2	3	4
1	0	100	150	0
2	120	0	160	0
3	135	140	0	0
4	0	0	0	0

Table 7. Time Limit

DC	Time Limit
1	7.00
2	7.00
3	7.00
4 ( <i>hub</i> )	23.00

The sensitivity analysis was performed with several parameters, such as usage cost of small and big vehicles, gas cost, and traveling time. The parameters were selected based on their effects on the model. The usage cost of small and big vehicles affected the route and the number of vehicles used. The traveling time affected the route and number of vehicles needed. The longer traveling time could increase the number of vehicles due to time window constraints. The sensitivity analysis was carried out for each parameter with change values of -20%, -10%, +10%, and +20%. One change parameter was used in one run. The case was solved using LINGO 1.0x64 bit.



### 3. Results and Discussion

#### 3.1 Implementation

The model was implemented in a case consisting of three DCs, eight customers, and four suppliers. The case needed 9 hours and 37 minutes of running time. The result can be seen in Fig. 1. The total cost generated for the routes is IDR 4,488,905.00.

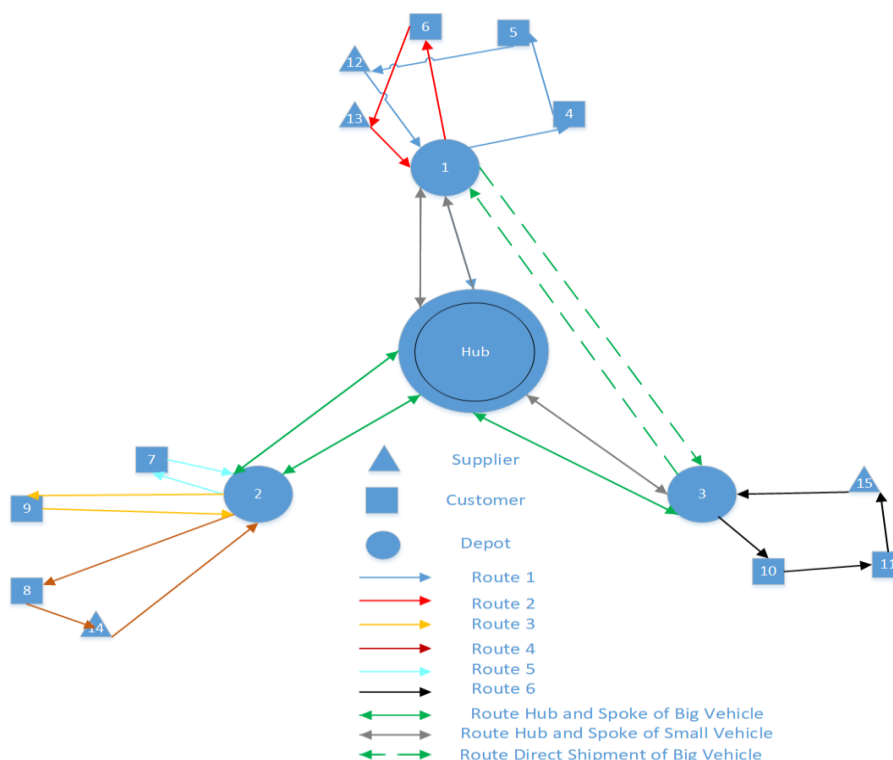


Fig. 1 The Solution Representation

#### 3.2 Sensitivity Analysis

The Implementation of sensitivity analysis was carried out to determine the change in the solution that occurred when the input of the parameters was changed. Sensitivity analysis was performed by changing certain parameters. Several parameters were changed. The modified parameters were vehicle usage cost for small vehicles, vehicle usage cost for big vehicles, gas cost, and traveling time. Before commencing the sensitivity analysis, the researchers operated the program several times with the change of parameters. The result of sensitivity analysis is given in Table 8.

The researchers found that the increase in the usage cost of small vehicles is followed by an increase in the total cost needed. However, the number of small and large vehicles used does not change. Meanwhile, there is also a change in the number of small and large vehicles where significant vehicle usage decreases and the number of small vehicles increases. An increase follows the increase in the usage cost of big vehicles in the total costs. The substitution of one large vehicle also follows it into two small vehicles as they have a lower cost. While the decrease in the cost of using large vehicles only reduces the total cost, the number of vehicles used is still the same.

Changes in fuel costs per km have the most significant impact as compared to other parameters. Increasing fuel cost per km by 20% raises an increase in the total costs of

8.57%. Also, a decrease in fuel cost per km by 20% causes a reduction in the total costs by 12.58%. It proves that the Indonesian government's policy on fuel prices has a considerable influence on transportation costs. Changes in traveling time affect the solution if the traveling time increases by 20%. Suppose the traveling time increases by 20% to satisfy the time windows. In that case, the number of significant vehicles needs to be added to cut the traveling distance of another vehicle. From all the factors, the fuel cost has the highest sensitivity to the solution. The distinction of this model with the previous model is that this model generates a route that makes sure all the vehicles arrive at each DC before the time limit.

Table 8. Sensitivity Analysis

Vehicle Usage Cost of Small Vehicle				
Percentage	Total Cost (IDR)	Quantity of Big Vehicle	Quantity of Small Vehicle	Delta cost %
20%	4,670,865	3	9	4.05%
10%	4,568,905	3	9	1.78%
0	4,488,905	3	9	0.00%
-10%	4,252,780	2	11	-5.26%
-20%	4,101,500	2	11	-8.63%

Vehicle Usage Cost of Big Vehicle				
Percentage	Total Cost (IDR)	Quantity of Big Vehicle	Quantity of Small Vehicle	Delta cost %
20%	4,597,780	2	11	2.43%
10%	4,553,780	2	11	1.45%
0	4,488,905	3	9	0.00%
-10%	4,422,905	3	9	-1.47%
-20%	4,353,905	3	9	-3.01%

Gas Cost				
Percentage	Total Cost (IDR)	Quantity of Big Vehicle	Quantity of Small Vehicle	Delta cost %
20%	4,873,686	3	9	8.57%
10%	4,636,296	3	9	3.28%
0	4,488,905	3	9	0.00%
-10%	4,161,515	3	9	-7.29%
-20%	3,924,124	3	9	-12.58%

Traveling Time				
Percentage	Total Cost (IDR)	Quantity of Big Vehicle	Quantity of Small Vehicle	Delta cost %
20%	4,713,050	4	9	4.99%
10%	4,488,905	3	9	0.00%
0	4,488,905	3	9	0.00%
-10%	4,488,905	3	9	0.00%
-20%	4,488,905	3	9	0.00%

This research only considers two cost factors, that is, vehicle usage cost and transportation cost. Many other cost factors may appear in this case, such as waiting time cost, late delivery/ pick up cost, and material handling cost. This model only allows customers to be served by one vehicle; if it is allowed to use more than one vehicle, it may enhance the vehicle utility and reduce costs.

#### 4 .Conclusion

This research aims to develop a model to determine the route for hybrid hub-and-spoke with time windows. The researchers developed a hybrid hub-and-spoke model with time windows to determine the inbound and outbound distribution routes. This model was designed using the Vehicle Routing Problem with Backhaul and Time Windows (VRPBTW) model and Hybrid hub-and-spoke as the basis for the modeling. Modifications are added to calculate time limits in the hub, calculation of vehicle departure time from the depot. Several additional constraints, such as sub tour elimination constraints, route restrictions, and other constraints, are also added. The results show that the model can solve the problem correctly. The suggestion For further research is to develop a model that allows a multi-pickup on suppliers and implement it in the actual case.

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