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# Geometric thinking program to enhance problem-solving skills of primary school teacher candidates

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**Abstract:** Geometry is a mathematics subject that has been widely considered hard. Several problems in learning geometry lead to geometric problem-solving skills. After exploring various causes, theories, and processes of geometric thinking, there should be an evaluation program to optimize problem-solving skills. This study aims to (1) develop a geometric thinking evaluation program and (2) enhance geometric problem-solving skills observed from the students' thinking level. The research and development adopted the Analysis, Design, Development, Implementation, and Evaluation (ADDIE) model, which started from the analysis stage involving four lecturers and 306 of students. The research was conducted at Primary School Teacher Education Department, Universitas Negeri Semarang. The process continued to the design stage, at which the evaluation program was drafted, followed by the development stage to test the validity. The last two phases included the trials in which the t-test and n-gain were applied to measure the program's effectivity and the evaluation phase aimed at assessing its practicality and the student's achievement in geometry. The established evaluation program covers an instrument of 39 multiple-choice items as an early assessment to determine the thinking level and learning trajectories as a scaffolding for improving geometric thinking skills. The trajectories, in the form of worksheets, were adjusted to the students' thinking level and were effective in assisting them in elevating to the informal reasoning level as well as refining their problem-solving skills.

**Keywords:** evaluation program, problem-solving skills, thinking geometric

## 1. Introduction

Geometry is a branch of mathematics concerned with properties and relationships between points, lines, planes, and space (Bergstrom & Zhang, 2016). It is learned at the primary to university level and is one of the most pivotal studies in mathematics (Anwar et al., 2023; Muzaini et al., 2023; Pavlovičová et al., 2022; Santos & Barbosa, 2023; Syam et al., 2020; Trimurtini et al., 2022). Geometry also widely studied across the world like Africa (Akayuure et al., 2016; Armah & Kissi, 2019), America (Bergstrom & Zhang, 2016), and Asia (MdYunus et al., 2019; Prayito et al., 2019; Syamsuddin, 2019).

Geometric thinking levels highly influence the success of studying geometry (Bonyah & Larbi, 2021; Škrbec & Čadež, 2015). These five include visual, descriptive, informal deduction, formal deduction, and rigor. Van Hiele believes these levels are governed by learning rather than age, attendance level, or biological maturity (Škrbec & Čadež, 2015). Hence, teachers need to be aware of what level each student has reached (Mammarella et al., 2017).

Geometry has been popularly believed to be a hard subject in mathematics. Several issues found in learning geometry are (1) perception problems (Özerem, 2012), (2) conceptual, principal, and procedural problems, (3) misunderstandings about visual processes and activities, (4) problems in using axiomatic deductive, (5) weak connections and reasoning in geometry (Susanti & Kurniawan, 2020). Overall, the main difficulties include often failing to understand geometric concepts and acquire geometric problem-solving skills (Armah & Kissi, 2019).

A geometric skills are observable through the thinking levels which are also the focus of this study (Casanova et al., 2021; Naufal et al., 2021; Ordiz & Mecate, 2022; Pavlovičová et al., 2022; Pavlovičová & Bočková, 2021; Vieira & Cyrino, 2022). The visualization level is characterized by recognizing the names, shapes, and characteristics of geometric objects (Hiele, 1999; Ordiz & Mecate, 2022; Pavlovičová et al., 2022; Ponte et al., 2023), while the descriptive level is marked by an understanding of the properties and relationships between geometric objects. The informal level is seen from the ability to formulate and use definitions (Casanova et al., 2021; Kandaga et al., 2022; Pavlovičová & Bočková, 2021; Walle et al., 2016). The formal level of reasoning is identified by the ability to construct evidence logically, clarifying the role of definitions and axioms (Casanova et al., 2021; Naufal et al., 2021; Uygun, 2022), and meta-mathematics is characterized by abstracting geometry without concrete models and comparing it with different geometric systems (Hohol, 2019). Moreover, several studies on geometric thinking levels with prospective mathematics teachers as the subjects showed that most students were at third level (Nopriana et al., 2023) and hardly ever reached highest level.

The description spells out this study's objective to develop a geometric thinking evaluation program to refine geometric problem-solving skills regarding thinking levels. The urgency is to discover the root cause of students' low problem-solving abilities and find a solution. Initially, Van Hiele determined the levels of geometric thinking from level zero to fourth. However, several subsequent studies changed from first level to fifth. Van Hiele's five levels of geometric thinking are visual logic, descriptive, theoretical, formal, and rigor (Bulut & Bulut, 2012; Hohol, 2019). Among these five stages, Indonesia's future teachers generally reach third level, which needs more (Bulut & Bulut, 2012; Nopriana et al., 2023). Thus, they shall be given the opportunity to soar their abilities.

Problem-solving skills can sharpen a students' thinking and reasoning by focusing on the process. It is also part of mathematics at all levels of education (Hadi et al., 2023). Challenges increase when problem-solving in geometry is presented as it involves a thorough comprehension of problems, geometric processing, various concepts, facts, and their applications (Novak & Tassell, 2017). An appropriate evaluation program for geometric thinking is needed to improve geometric problem-solving skills.

Based on the background of geometry learning problems and the importance of problem-solving abilities, the aims of this research are to (1) develop a geometric thinking evaluation program and (2) enhance geometric problem-solving skills observed from the students' thinking level.

## 2. Materials and Methods

### *2.1 Research Procedure*

This is a developmental study adopting the ADDIE model. The stages include Analysis, Design, Development, Implementation, and Evaluation (Figure 1). The research was conducted for 10 months at the Department of Primary School Teacher Education (*Pendidikan Guru Sekolah Dasar*/PGSD), Faculty of Educational Sciences and Psychology, Universitas Negeri Semarang, Central Java - Indonesia. As many as 306 students in a fourth-semester were participated in this research. In detail, this developmental research is described in the activity flow chart in Figure 2.

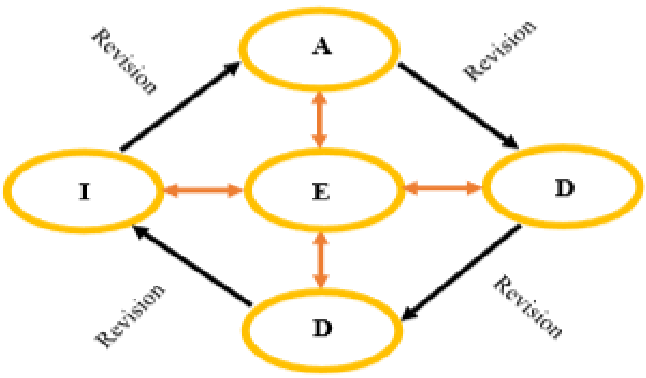

Figure 1. The ADDIE model in developmental research (Muruganantham, 2015)

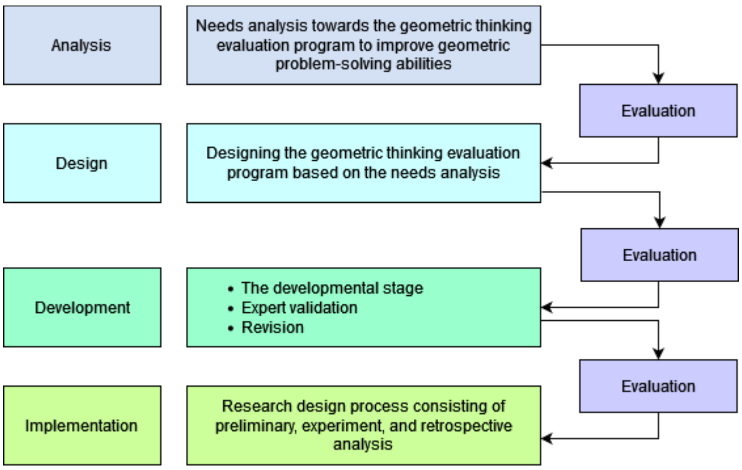

Figure 2. Stages of developmental research with the ADDIE model

An analysis of the geometric thinking evaluation program is needed to improve problem-solving ability. Three hundred and sixteen students were involved in filling out the needs analysis questionnaire. Four mathematics lecturers participated in Focus Group Discussion (FGD) to discuss problem-solving skills related to geometric thinking.

The research team designed a geometric thinking evaluation program based on the need analysis questionnaire and FGD results. At the development stage, the design was assessed by the validators. The research team evaluates and revises the design based on the validator's suggestions. Later than, the geometric thinking evaluation program is implemented in one class. This stage uses a research design process consisting of preliminary, experiment, and retrospective analysis (Creswell, 2014). The problem-solving skills were tested in the experimental and control classes, and the results were analyzed.

### *2.2 Data Analysis*

There were two data analyses carried out in this research, which included analysis of feasibility and responses to the geometric thinking evaluation program. The instrument for assessing the program feasibility and the responses of lecturers and students were analyzed using the formula (1):

$$P = \frac{f}{N} \times 100\% \qquad (1)$$

Information:

P = percentage

f = gained score

N = overall score

The effectiveness was measured with a proportional test on the achieved minimum completeness, an experimental-class comparison test, and an n-gain test for the increase of geometric problem-solving skills. The minimum score was 81, accomplished by 75% of the students.

## 3. Results

### *3.1 Analysis*

At the needs analysis stage, the students were given geometry learning and assessment questionnaires. The results are as follows in Figure 3.

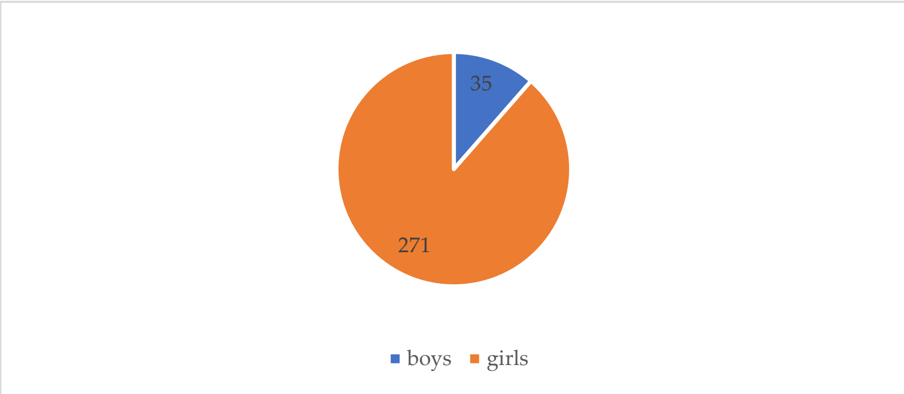


Figure 3. Comparison of the number of students by gender

The proportion of fourth-semester Primary School Teacher Education Department students was 88.6% female and 11.4% male. This shows that women are more interested in becoming prospective elementary school teachers.

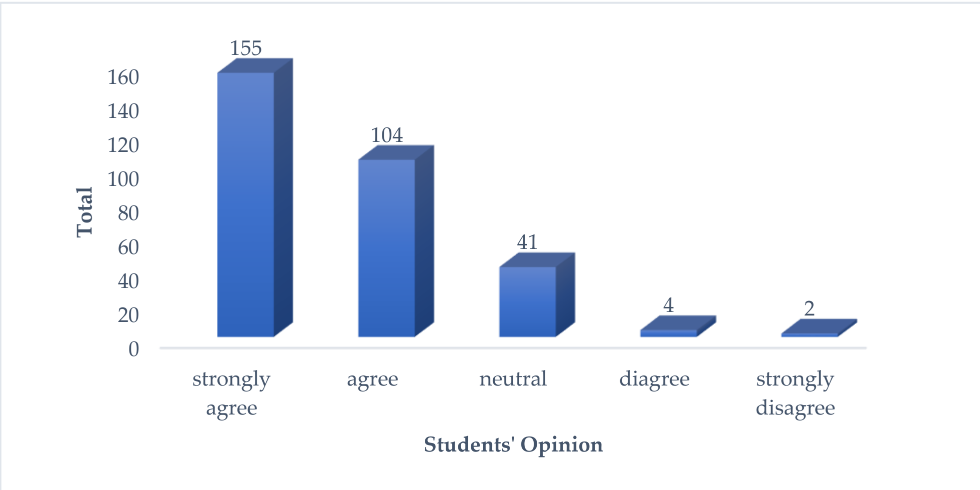


Figure 4. Students' opinions about the use of geometry in life

The majority of students (50.65%) agreed that geometry is useful and related to everyday life (Figure 4).

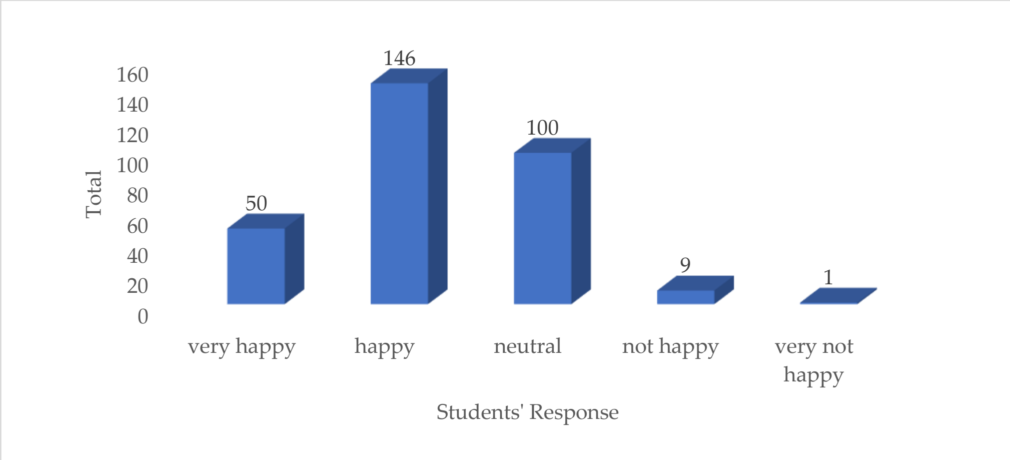


Figure 5. Sentiments of studying geometry

Students showed enjoy in learning geometry (64.05%) (Figure 5), yet some remained unhappy; one even did not enjoy it at all. Further investigation revealed that those having a hard time in geometry had not experienced a pleasant learning environment, either at school or when studying at Primary School Teacher Education Department.

### 3.2 *Design*

The design was created based on the results of the needs analysis. The evaluation program begins with composing instruments and hypothetical learning trajectories. The developed instrument is the Geometric Thinking Level Test (GTLT). The initial abilities of geometric thinking were measured using a test instrument with multiple-choice questions created by adapting previous similar research. The GTLT materials cover flat geometry, which is the most troubled topic at elementary department.

The developed GTLT refers to the operational definitions and indicators of geometric thinking levels resulting from a systematic literature review (Nopriana et al., 2023; Trimurtini et al., 2022). Initially, as many as 39 questions were developed with details as in Table 2.

Table 2. The GTLT outline

| No | Geometric Thinking Level | Indicator | Item Number |
|---|---|---|---|
| 1 | **Introduction:** Identifying geometric shapes and objects. | a. Showing geometric shapes and objects. | 1*, 2, 3 |
| | | b. Distinguishing between the shapes of one geometric object and another | 4, 5, 6 |
| | | c. Grouping geometric objects based on their shape | 7, 8 |
| 2 | **Descriptive:** Identifying the properties of geometric objects and the relationship between objects | a. Identifying the properties of geometric objects | 9, 10, 11, 12 |
| | | b. Stating the relationship between geometric objects based on the properties | 13, 14 |
| | | c. Distinguishing geometric objects based on the properties | 15, 16 |
| | | d. Grouping geometric objects based on the similarity of properties | 17*, 18 |
| 3 | **Informal Reasoning:** Formulating and using definitions, differentiating between necessary and sufficient conditions in a set of geometric object properties. | a. Analyzing the relationship between the properties of geometric objects | 19, 20, 21 |
| | | b. Analyzing the definition of geometric objects | 22, 23, 24 |
| | | c. Analyzing necessary and sufficient conditions in a question | 25*, 26, 27 |
| 4 | **Formal Reasoning:** Constructing proofs logically, clarifying the role of definitions and axioms, and justifying each step in the proof. | a. Proving statements | 28, 29, 30 |
| | | b. Evaluating the reasons behind the steps of proof | 31*, 32, 33 |
| 5 | **Meta-mathematics:** Abstracting geometry without concrete models; | a. Evaluating the overall proof process | 34, 35* |

| No | Geometric Thinking Level | Indicator | Item Number |
|---|---|---|---|
| | establishing theorems in different geometric systems by comparing them with existing ones. | b. Finding the relationship between definitions and theorems<br>c. Building theorems in different geometric systems | 36*, 37<br>38, 39 |

Note: * = invalid question.

Experts assessed the GTLT 's content validity by evaluating the suitability of the operational definition, indicators, and questions to indicators, language aspects, and work instructions. The four experts are lecturers of Mathematics Education at Universitas Negeri Semarang, while the others is a geometry specialist lecturer of Mathematics Education at Universitas Sebelas Maret. Determining the achievement category for each indicator is based on a percentage distribution of 0%-25% in the very poor category, 26%-50% in the poor category, 51%-75% in the good category, 76%-100% in the very good category.

Table 3. The results of experts' validation

| No | Indicator | Ideal Score | Actual score | | | | | Average | NP (%) | Category |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Expert 1 | Expert 2 | Expert 3 | Expert 4 | Expert 5 | | | |
| 1 | Suitability of the operational definition | 20 | 18 | 16 | 20 | 16 | 16 | 17,2 | 86 | Very good |
| 2 | Suitability of aspects and indicators of geometric thinking levels | 30 | 24 | 24 | 28 | 25 | 26 | 25,4 | 84.67 | Very good |
| 3 | Work instructions | 10 | 9 | 8 | 10 | 8 | 9 | 8,8 | 88 | Very good |
| 4 | Question item | 35 | 29 | 28 | 31 | 28 | 28 | 28,8 | 82.28 | Very good |
| 5 | Language | 15 | 13 | 13 | 15 | 13 | 14 | 13,6 | 90.67 | Very good |

Based on the expert validation results (Table 3), the GTLT questions scored 86.32% in a very good category. The GTLT was designed with 39 multiple-choice questions, and after being revised according to expert suggestions, it went to further validity and reliability tests. The GTLT trial showed that six questions were invalid for numbers 1, 17, 25, 31, and 35, while the other 33 were valid. Meanwhile, the reliability reached 0.577 (quite reliable). In other words, the 33 GTLT questions were declared valid, reliable, and worthy of use.

The GTLT scoring guidelines and determination of geometric thinking levels were taken from previous studies (Nopriana et al., 2023; Trimurtini et al., 2022), which confirmed that achievement is indicated by 60% to 80% correct answers. In this study, accomplishing >75% correct answers means promotion to the next level, while the highest level of meta-mathematics is reached upon having 75% correct answers. A total of 33 questions were distributed at each level, as in Table 4.

The data on the geometric thinking level were collected through tests. There were 13 classes with a total of 306 students working on the GTLT, and each student was assessed on which level students was in. These data were beneficial for determining the research subjects at each level. Two to three students who had agreed to be research subjects were selected to continue to the analysis stage.

Table 4. Guidelines for determining the geometric thinking level

| No | Level | Number of Question | ≥ 75% correct answers for levelling up |
|---|---|---|---|
| 1 | Introduction | 7 | 5 |
| 2 | Descriptive | 9 | 7 |
| 3 | Informal Reasoning | 8 | 6 |
| 4 | Formal Reasoning | 5 | 4 |
| 5 | Meta-mathematics | 4 | 3 |
| | **Total of Question** | **33** | |

### 3.3 Development

The development stage was carried out after the instrument was declared valid and reliable, followed by the validation of hypothetical learning trajectories done by two experts (Table 5). The Hypothetical Learning Trajectory for Geometric Thinking (HLTGT) was arranged based on the students' early knowledge and thinking. The results of the analysis regarding the comparison of geometric thinking process patterns at each level have led to scaffolding guidelines. The developed HLTGT is a tool to promote geometry learning so that students can increase their level of geometric thinking. Components in HLTGT include learning objectives, learning activities, and a cognitive model called progressive development. The HLTGT's flat geometry materials are adapted to student needs, especially in (1) basic geometry, (2) triangles, (3) quadrilaterals, and (4) circles. Learning Trajectory Geometric Thinking (LTGT) was produced from the HLTGT through a research design process consisting of preliminary, experiment, and retrospective analysis (Creswell, 2014).

The preliminary stage designed HLTGT based on the analysis results of geometric thinking process patterns at each level and literature studies on geometric thinking. The prepared HLTGT was then assessed for suitability by two experts, and the results are as follows in Table 5.

Table 5. The HLTBG's Validation Results

| No | Indicator | Ideal Score | Actual Score | | Average | NP* (%) | Category |
|---|---|---|---|---|---|---|---|
| | | | Expert 1 | Expert 5 | | | |
| 1 | Usage instruction of learning trajectory | 10 | 10 | 8 | 9 | 90 | Very Good |
| 2 | Content of learning trajectory | 30 | 28 | 26 | 27 | 90 | Very good |
| 3 | Language | 15 | 12 | 13 | 12.5 | 83.33 | Very good |

The feasibility was assessed in terms of language, suitability of content, and usage instructions. The HLTGT was rated in the very good category (87.78%). After adjustments were made, the scaffolding guidelines became more appropriate to students' initial abilities and helped overcome learning difficulties (Desmayanasari & Hardianti, 2021). The scaffolding guidelines, in the form of a learning trajectory, underwent some revision based on the experts' feedback, primarily for combining it with the HLTGT with some explanations and reviews (Anghileri, 2006) on the concepts and properties of geometry. This change is beneficial, especially for students at the introductory and descriptive levels.

The next phase was the trial, which consisted of teaching and pilot experiments. The teaching experiment explored the students' prior knowledge and collected data to support adjustments to the learning trajectory. There were 38 students in the experimental class and 38 students in the control class. The majority of the students were at the introductory level, and only a few reached the meta-mathematics level. The HLTGT components include instructions for use, geometric thinking level components, learning objectives, and learning activities.

### 3.4 Implementation

The geometric thinking evaluation program was applied in the experimental class, while the control class ran the usual learning activities using a geometry learning handbook, group discussion, and presentation. Samples were taken for the experimental and control classes using the cluster random sampling technique, and group F (38 students) was taken as the experimental class. In contrast, group L (38 students) was taken as the control class. After the treatment, students worked on geometry problem-solving questions.

The effectiveness of the geometric thinking evaluation program was analyzed in several stages. First, the completeness of problem-solving skills in the experimental class reached a value of 81 at 75%. Second, in the experimental class, there was an increase in geometric thinking skills. Third, the problem-solving ability in the experimental class was better than the control class. The following presents the data analysis results.

*3.4.1 Completeness Test*

The completeness test was intended to determine whether or not the problem-solving skills of students who experienced learning using the learning trajectory had reached the predetermined minimum completeness criteria. In this case, the minimum score for geometry material was 81, with 75% classical completeness. The hypotheses tested for individual completeness were:

$H_0: \mu \leq 81$ (The average of students' problem-solving skills ≤ 81)

$H_1: \mu > 81$ (The average of students' problem-solving skills > 81)

The completeness test can be calculated following the one-sample t-test presented in Table 6.

Table 6. Test of learning completeness

| t-count | df | Sig. |
|---|---|---|
| 5.204 | 37 | 0.000 |

The results showed that a sig value is < 0.05; thus, $\boldsymbol{H_0}$ was rejected, and the students' average score of problem-solving skills was> 81. The classical completeness test was performed after carrying out the individual completeness test. This test employed a proportion test with H0 rejection if $\boldsymbol{z} \geq \boldsymbol{z_{0,5-\alpha}}$ where $\boldsymbol{z_{0,5-\alpha}}$ was obtained from the standard normal list with a probability of $(\boldsymbol{0,5-\alpha})$. The proposed hypotheses were:

$\boldsymbol{H_0}: \pi \leq \mathbf{75\%}$ (The proportion of students' problem-solving skills accomplishing the minimum completeness is less than or equal to 75%)

$\boldsymbol{H_1}: \pi > \mathbf{75\%}$ (The proportion of students' problem-solving skills accomplishing the minimum completeness is more than 75%)

The proportion test to calculate the completeness score was carried out manually using the following formula:

$$z = \frac{\frac{x}{n} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

$$z = \frac{\frac{33}{38} - 0{,}75}{\sqrt{\frac{0{,}75\,(1-0{,}75)}{38}}}$$

$$z = 1{,}686$$

From the list of standard normal tables, the results were $\boldsymbol{z_{0,5-\alpha}}$ or $\boldsymbol{Z_{table}}$ of 1.64. Because $\boldsymbol{Z_{count}} > \boldsymbol{z_{table}}$ or 1.686 > 1.64, then $\boldsymbol{H_0}$ was rejected. It concluded that the proportion of students' problem-solving skills after using the learning trajectory met the minimum completeness criteria by more than 75%.

*3.4.2 The Increase of Geometry Learning Results*

The increase in geometry learning outcomes was measured using the paired-samples t-test to see the differences between the two classes before and after the learning trajectory was applied. The proposed hypotheses were:

$\boldsymbol{H_0}: \boldsymbol{\mu_1} \geq \boldsymbol{\mu_2}$, There is no difference in geometry learning outcomes before and after using the learning trajectory

$\boldsymbol{H_1}: \boldsymbol{\mu_1} < \boldsymbol{\mu_2}$, There is an increase in geometry learning outcomes before and after using the learning trajectory

A difference test was performed with $H_0$ acceptance if sig > 0.05. The outputs of the difference test are presented in Table 7.

Table 7. The pretest and post-test results of geometry learning

| Data | T count | Sig | Conclusion |
|---|---|---|---|
| pre-post experiment | -5,110 | 0,000 | Significant difference |

Based on the difference test on SPSS 23 using the paired sample test with a 0.05 significance level, the significance value for the experimental class was 0.000 < 0.05. Thus, $H_0$ was rejected. The conclusion is that there is a significant difference between the pre-test and post-test in the experimental class. This indicated an increase in geometry learning outcomes by 15%, meaning that using the geometric thinking evaluation program in the form of a learning trajectory positively influenced the students' geometry learning outcomes.

*3.4.3 Comparison Test*

The comparison test was to determine the difference between the student's problem-solving skills in the experimental class and those in the control class. The comparison test was carried out using the independent t-test, and the proposed hypothesis was:

$\boldsymbol{H_0}: \boldsymbol{\mu_1} \leq \boldsymbol{\mu_2}$ (no significant difference found in the student's problem-solving skills in the two classes)

$\boldsymbol{H_1}: \boldsymbol{\mu_1} > \boldsymbol{\mu_2}$ (the experimental class students' problem-solving skills are better than those of the control class)

The test criterion was to reject Ho if t count > t table or Sig value <0.05. The results of the independent sample t-test showed a sig value. of 0.000 < 0.05, meaning that there was a significant difference in the problem-solving abilities of experimental and control class students. Due to this finding, a further test was conducted by looking at the average test score.

The average score of problem-solving skills by the experimental class students was 89, higher than those of the control class students, who reached 79.3. The increase in students' thinking levels was measured using the geometry test, which produced categories of thinking levels before and after the treatment. The results are presented descriptively in Figure 6.

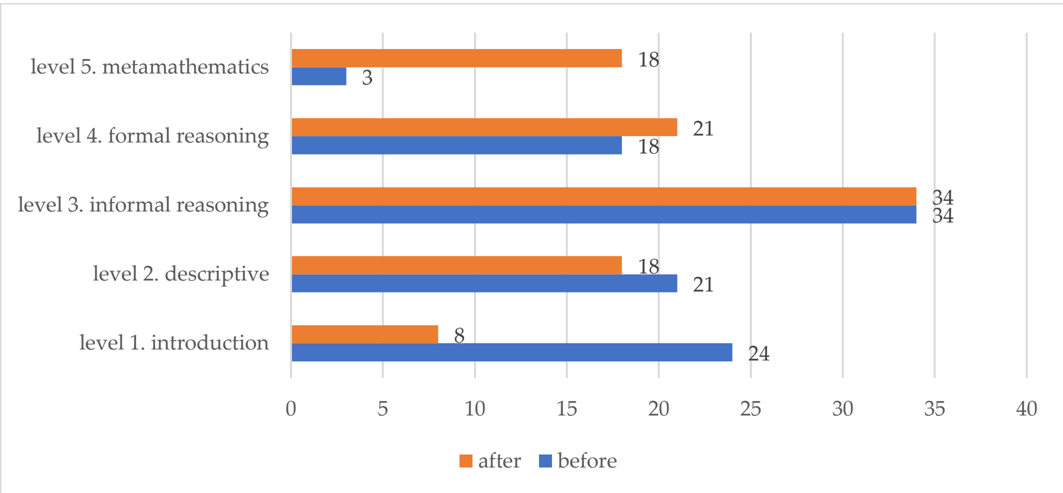


Figure 6. The increase in geometry thinking level in the experimental class

The students' levels of thinking are divided into five categories: introductory, descriptive, informal reasoning, formal reasoning, and meta-mathematics. Based on the results in Figure 6, the percentage of students were more likely to be categorized as introductory, descriptive, and informal reasoning before learning. Meanwhile, after the treatment, the student's level of thinking increased to informal reasoning, formal reasoning, and meta-mathematics. These results explain that learning using scaffolding guidelines in the form of a learning trajectory can increase students' level of thinking.

## 4. Discussion

The development of a geometric thinking evaluation program for elementary pre-service students which consists of an initial assessment to find out the level of geometric thinking, helps students realize the conditions and potentials they have in learning geometry. This aims to gradually improve the level of geometric thinking as well as their ability to solve geometric problems. Teacher candidate students should reach a minimum of third level (informal reasoning) (Andini et al., 2018; Armah & Kissi, 2019; Bulut & Bulut, 2012) so that he has the bearing capacity to master geometry material and deepen it with appropriate geometry learning strategies. The evaluation program developed and the implementation of learning through HLTGT showed that Primary School Teacher Education Department students were able to improve from two first level to a higher level (Figure 5). Indicators of increasing levels of geometric thinking that have been analyzed, most students not only know the properties of geometric shapes, but can relate their relationships and use them to prove a statement or to solve geometric problems. At level formal reasoning, students also understand the role of definitions, theorems, axioms, and proofs (Hohol, 2019).

The HLTGT components include instructions for use, geometric thinking level components, learning objectives, and learning activities that are also effective in improving geometry problem-solving skills. Various problems of learning student geometry can be overcome by (1) perception problems (Özerem, 2012), (2) problems in the use of concepts, principles, and procedures, (3) misconceptions about visual processes and activities, (4) problems in the use of axiomatic deductive, (5) weak connections and reasoning in geometry (Susanti & Kurniawan, 2020). Problems of perception, use of concepts, principles, and procedures in geometry can be overcome with HLTGT, when students know their level of geometric thinking, so that they are given learning assistance that is appropriate to their level, and the material provided is adjusted to their level of geometric thinking. Students' weaknesses in connection and geometric reasoning can also be solved by the learning stages presented in HLTGT. The scaffolding guidelines, in the form of a learning trajectory, underwent some revision based on the experts' feedback, primarily for combining it with the HLTGT with some explanations and reviews (Anghileri, 2006) on the concepts and properties of geometry. This change is beneficial, especially for students at the introductory and descriptive levels. This aligns with similar research on learning trajectory that has been declared worthy of improving geometry skills (Budhayanti & Bata, 2021; Hadila et al., 2020; Zuliana, 2017). The learning trajectory enables students to learn according to their level to experience improvement and reach the next level.

Furthermore, the ability to solve geometry problems in Primary School Teacher Education Department students was initially very low. Students are not used to working on geometry problem-solving problems. So some of the difficulties faced are due to the lack of a basis for geometry concepts, difficulties in perception, and geometric reasoning (Susanti & Kurniawan, 2020). Through the implementation of learning trajectory, students who reach the meta-mathematical level can work on solving geometry problems in sequence, accompanied by reasoning based on concepts and even theorems in geometry. This is as shown in the results of the students' answers as shown in Figure 6.

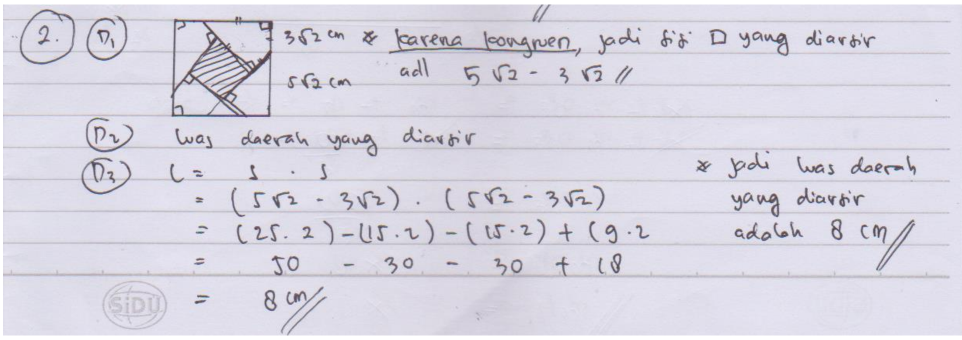

Figure 7. Stages of solving geometry problems in students at the formal reasoning level

The students' answers in Figure 7 show that students can visually understand the components of a two-dimensional figure, including shapes, sides, and angles. Furthermore, students identify the similarities in the two-dimensional figure and conclude several congruent shapes. To solve the problem of calculating the shaded flat plane, students can relate it to the formula for the area of a square. Students at third level (formal reasoning) and fourth level (meta-mathematics) are expected to have advanced problem-solving skills, as they can logically create evidence and prove axiomatic theorems, respectively (Hendriyanto et al., 2021; Mawarsari et al., 2023).

## 5. Conclusion

The developed geometric thinking evaluation program consists of a multiple-choice test as an initial assessment to determine the level of geometric thinking. The levels of geometric thinking are introduction, descriptive, informal reasoning, formal reasoning, and meta-mathematics. This is followed by scaffolding in the form of a learning trajectory following the students' thinking level. This learning trajectory effectively increases the level of geometric thinking upon reaching informal reasoning and improving problem-solving skills. The learning trajectory components include instructions for use, geometric thinking level components, learning objectives, and learning activities.

**Authors Contribution:** Trimurtini: methodology, conducting the research and writing original article, field data collection, data analysis, and revision. N. Nugraheni: field data collection and data analysis. B. Waluya: methodology and supervision. E.F. Sari: field data collection and editing article.

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