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Learning obstacles in the generalization process: In case number pattern topic

Nyla Farhatul Maula[a,1], **Didi Suryadi**[a,2,*], **Al Jupri** [a,3]

[a] Department of Mathematics Education, Faculty of Mathematics and Sciences Education, Universitas Pendidikan Indonesia, Jl. Setiabudi No. 229, Isola, Sukasari, Bandung, West Java 40154, Indonesia
[1]nylafara@upi.edu; [2]didisuryadi@upi.edu*; [3]aljupri@upi.edu
*Corresponding author

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Abstract: Generalization plays an important role in mathematics because it is considered inherent in mathematical thinking in general. Where number pattern is a topic that is closely related to generalization. Problems are still found in generalizing number patterns so this can cause learning obstacles. This research aims to identify learning obstacles for Junior high school students in the generalization process with a focus on number pattern topics. This research is qualitative research with a case study method. The research subjects consisted of 30 grade 8 students in Jakarta who had studied number pattern. Subjects were given three test tasks, the results of which were used to figure out learning obstacles in the generalization process experienced by students and then continued with interviews with ten subjects. The findings show that learning obstacles in the pattern generalization process occur primarily at the expression and symbolic stages. Most students bypassed the generalization method, relying instead on formulas or manual calculations. Based on these results, it is recommended that educators implement didactic designs that offer targeted interventions during the expression and symbolic stages, such as using more interactive and symbolic reasoning activities to strengthen students' understanding and generalization skills in number patterns.

Keywords: learning obstacles; generalization; number pattern

1. Introduction

Generalization is the recognition of several common characteristics in a set of mental objects (Dreyfus, 1991). Generalization is also defined as the process of finding similarities or patterns in each example or case so that the general order can be applied (Brief, 2003). Generalization is not a concept but a procedure (Radford, 2001). It can be concluded that generalization is not just a technical procedure, but also a fundamental cognitive process in understanding regularities and patterns.

To find out how the generalization process occurs, there are four stages in this generalization process. According to Marzano (1988) the generalization process stage is divided into 4 namely: (1) Perception of generality namely the process of perceiving or identifying patterns; (2) Expression of generality namely determining the structure or data or picture or next term from the results of findings or pattern identification; (3) Symbolic expression of generality namely formulating generality symbolically; (4) Manipulation of generality namely solving problems using generalization results.

Perception of generality includes an individual's understanding of general patterns or rules that can be applied to various series of numbers or certain sequences. At this stage, students recognize and understand the structure or rules that underlie patterns in number series. Expression of generality involves a person's ability to communicate or apply general rules or patterns in matters that follow. Symbolic of generalization involves the use of symbols or mathematical representations to describe general rules or patterns in number series. Manipulation of symbols involves a person's ability to perform mathematical manipulations on symbols or algebraic formulas that describe general rules. Understanding these stages helps to see why generalization is crucial in mathematics education.

In general generalization plays an important role in mathematics because it is considered inherent in mathematical thinking in general (Stacey, 1989; Dindyal, 2017; Barbosa & Vale, 2015). The important role of generalization in mathematics has led several researchers to conduct research on generalization. Consequently, several researchers have focused on generalization. Research on generalization often explores various mathematical topics. Number patterns are the most frequently researched material regarding generalization (Dindyal, 2007; Fadiana, 2018; Guner et al., 2013; Kusumaningtyas et al., 2017; Somasundram et al., 2019).

Previous research related to the generalization of number patterns shows the use of several methods and approaches to help students recognize and solve mathematical patterns. One of the dominant methods is algebraic reasoning and symbolic representation. Several studies show that students use cross-check strategies and explicit strategies for generalization, whereas explicit strategies are more often used for distant generalization. In addition, students with an independent cognitive style are more likely to use an analytical approach in solving patterns, while students with a dependent cognitive style rely more on global observations of patterns. Another study highlighted the role of figural and numerical patterns in mathematics teaching where students were asked to connect between visual and numerical patterns. The use of semiotics is also an effective solution to help students generalize patterns, especially by using gestures, speech, and writing to visualize and express patterns. For gifted students, the Gauss approach in nonlinear pattern generalization is introduced as a more efficient way to handle more complex patterns. This approach is used to help gifted students develop deep functional thinking related to number patterns.

Among the various studies on number pattern generalization, there are two that make significant contributions and can be considered as main references. Güner et al. (2013) offer an in-depth understanding of the explicit strategies used by 7th and 8th grade students to perform far generalization. These strategies have been shown to be effective in helping students develop algebraic thinking patterns, especially when they are asked to extend numerical patterns into more complex terms. Using semi-structured interviews, this study provides important insights into students' thinking processes and helps explain how students can overcome difficulties in the transition from near to far generalization. These findings are highly relevant to support more effective teaching in developing students' ability to generalize number patterns.

Meanwhile, Yildiz and Durmaz (2021) focused on the use of the Gauss approach for generalizing nonlinear patterns. This approach provides an efficient solution for gifted students to handle more complex patterns quickly and accurately. The success of the Gauss approach in helping gifted students understand nonlinear patterns shows its potential for wider application, even beyond the scope of gifted students. This study emphasizes the importance of greater cognitive challenges in helping students overcome the barriers to understanding in the process of generalizing number patterns.

Although previous research provides significant insights into generalization strategies, there are several limitations that need to be addressed as research gaps. One major limitation is the lack of an in-depth explanation of the barriers that students face in the generalization process. Research tends to focus on the use of explicit strategies, but fails to explore in detail because students choose certain strategies or how they experience difficulties in implementing them. In addition, most studies focus only on numerical patterns, while figural patterns are less explored, limiting the applicability of research findings to more complex contexts. Research on generalization in gifted students has also shown the effectiveness of more sophisticated approaches, such as the Gaussian approach, but this is limited by the lack of attention to students with lower abilities. This leads to a gap in understanding how students with different levels of ability can be empowered to overcome barriers to generalization, especially in more complex patterns. These limitations point to the need for more comprehensive research on learning barriers to generalization, particularly by expanding the scope to include both numerical and figural patterns, as well as students with varying abilities.

Based on the limitations identified from previous studies, it is known that although many studies have explored generalization strategies, there is still a significant gap in understanding the specific learning obstacle faced by students during the generalization process, especially in the context of number patterns. Some researchers have focused on identifying the strategies used by students, such as explicit strategies and cross-checking, but studies that discuss in depth the learning obstacles underlying students' failure to generalize, especially across different types of patterns (numerical and figural) and different levels of cognitive ability, are still very limited. Therefore, this study aims to systematically identify and describe the learning obstacles faced by junior high school students in the generalization process on number pattern material. The purpose of this study is to uncover the learning obstacles that hinder students' ability to generalize, as well as provide insights that can help develop more targeted teaching strategies to support students in overcoming these obstacles.

This study attempts to provide a new perspective by directly addressing the gap in the literature related to learning barriers in the process of generalization on number pattern material. The scientific value of this study lies in its approach to providing a deeper understanding of the learning process by highlighting factors that hinder the process of generalization on number pattern material, especially in junior high school students. In addition, the results of this study will provide practical implications for educators, by equipping them with the knowledge to design more effective didactic designs that are directed at overcoming these barriers. By focusing on the process of generalization and learning barriers, this study is expected to fill an important gap in the field of mathematics education, both in theoretical aspects and practical pedagogical improvements.

2. Materials and Methods

2.1 Types of research

This research used a qualitative approach with a case study design in accordance with the research objectives. Instruments in the form of tests and unstructured interview guides were used in this research. The qualitative approach was chosen because it allows researchers to explore students' experiences and understandings in depth regarding the obstacles they face during the generalization process on number pattern material. The case study design is used to provide a special focus on real situations faced by students in the learning process so that they can explore the phenomena that occur in depth and contextually. This study seeks to thoroughly explore the obstacles that arise during the generalization process.

2.2 Research Subjects and Objects

The subjects in this research were 30 students from one class at junior high school level in Jakarta. The research subjects were class VIII students aged 13-15 years. A total of 30 students in the same class were given a test instrument consisting of three tasks. Students were given answer sheets to write in detail each method used to get the answer. The time for working on the questions was 60 minutes where when working on the questions students were not allowed to use calculators, handphones, or other tools and students work on the questions individually.

2.3 Sample Collection Techniques

For each answer given by students from the three tasks, the researcher analyzed and grouped them based on the similarity of the type of answer. From the results of this analysis, ten students were selected to conduct interviews with researchers, where these ten students were considered to represent all of the students' answers. The interview was carried out 7 days after the student took the test because it adjusted to the availability of the student's schedule at school. Each student interviewed for 5-10 minutes by answering questions asked by the researcher. The questions given are related to how students work on the questions given so that researchers can find out the obstacles faced by students.

2.4 Data Analysis Techniques Data Analysis Techniques

The research data were analyzed using data analysis techniques that divided into 5 phases. First compiling, arranging all data in a certain order. Data was gathered from 30 eighth-grade students in Jakarta who had previously studied number patterns. This included test results designed to identify learning obstacles in the generalization process and interviews with ten selected students for deeper insights. Second disassembling, breaking down the data obtained into smaller parts. Test results were analyzed to identify specific difficulties at different stages of the generalization process, and interview data was examined to further understand students' learning obstacles. Third reassembling, data that has been broken down previously reorganized to be able to answer research questions. By combining findings from tests and interviews, a comprehensive picture of the learning obstacles in the generalization process was formed. Fourth interpreting, interpreting the data to form a description of the data. The data was interpreted to form a detailed description of the identified learning obstacles. The analysis focused on understanding how perception, expression, symbolic, and manipulation stages contributed to these obstacles. Fifth concluding, the researcher making conclusions from the entire research. The findings highlighted the specific stages where students faced difficulties in generalizing number patterns, providing a foundation for developing more effective didactic designs to overcome these obstacles.

3. Results

To achieve the research objective, namely describing the learning obstacles of Junior High School students in pattern material with a focus on the generalization process. The research subjects were given a test containing three tasks to determine the learning obstacles experienced by students in carrying out the generalization process on pattern material.

Based on data obtained from three tasks taken by 30 students, the following are the results of the research which include student success rates, solution methods used, learning obstacles faced, and recommendations for teaching.

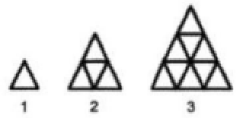
Analysis of student answer results is seen from the level of student success in answering the three tasks given. This analysis is presented in Table 1.

Table 1. Distribution of student success levels

Task	Number of Correct (%)	Number of Incorrect (%)	Number of Blank (%)
1	14 (46.67)	15 (50)	1 (3.33)
2	1 (3.33)	15 (50)	14 (46.67)
3	13 (43.33)	12 (40)	5 (16.67)

Table 1 shows the distribution of student success for each task, including the number of correct, incorrect, and blank responses, as well as the percentages for each category. From the table above, it can be seen that task 1 has a higher success rate than task 2 and 3. Task 2 has the highest failure rate with 50% of students got incorrect answer and 46.67% of students not answering (Figure 1).

1. Look at the following image. By looking at the existing pattern, how many triangles small at 50th? Also write down every step you take get those answers!



2. Look at the sequence below then determine the formula for the nth term of the sequence! Continue by finding the value of the 25th term of the sequence! Don't forget to write down every step you took to get the answer.

$$1, 5, 12, 22, \ldots$$

3. Pay attention to the rows below. How many series are there in the last row (which is marked with a square)! Also write down every step you took to get the answer.

$$2 = 2$$
$$2 + 4 = 6$$
$$2 + 4 + 6 = 12$$
$$2 + 4 + 6 + 8 = 20$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$2 + 4 + 6 + 8 + \cdots + 100 = \square$$

Figure 1. The tasks are given in the test

Data was collected from the results of students' answers and showed that there were several solution strategies used by students in answering these three tasks. The strategies used are grouped by researchers into 3, namely by generalizing, manual calculations, and using formulas. Table 2 shows the results of the distribution of the solution strategies used by students in each task item.

Table 2. Distribution of students' strategies

Task	Generalization (%)	Manual (%)	Formula (%)
1	14 (46.67)	13 (43.33)	1 (3.33)
2	1 (3.33)	14 (46.67)	3 (10)
3	3 (10)	22 (73.33)	0 (0)

From the Table 2, the manual strategy is most often used by students, especially in task 3 (73.33%). The generalization strategy was used more often in task 1 (46.67%). The use of formulas is very low and is not even used in task 3.

Considering the level of student success and the strategy used for each task, out of a total of 30 students the researcher chose to conduct interviews with ten students. Based on the analysis of student test results and interviews, the research results will be discussed for each question. This includes an explanation of the methods used, whether through generalizations, manuals, or formulas, and the learning obstacles that students go through in solving the problems presented.

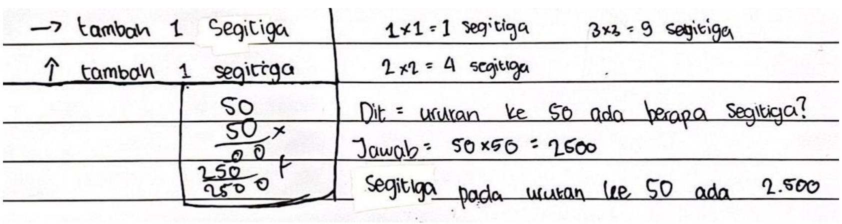
Generalization method involves students recognizing patterns and deriving a general rule or formula that applies to a broader set of numbers or cases. When using generalization, students try to find a commonality between different examples and then create a general expression or rule that can be applied to any case in a series. In this context, manuals method likely refers to manual calculations or step-by-step problem-solving methods. Some students, instead of generalizing, may rely on performing calculations for each specific case without attempting to find a broader rule. Formula strategy is an approach where students directly apply the formulas that have been taught to solve problems without going through the process of understanding patterns or generalizing.

In general, the following results of the analysis carried out based on the strategies and obstacles encountered are presented in Table 3, Table 4, and Table 5 for each question. Table 3 shows that there were three categories of students in task 1 along with the strategies and obstacles they experienced in solving the question using the strategy they used. Students who generalize with correct results still experience obstacles at the symbolic stage. Apart from that, obstacles were also found in manual calculations carried out by students and the selection of formulas that were not appropriate to the context of the problem being presented.

Table 3. Analysis of strategies and obstacles learning in task 1

Strategy	Description	Learning Obstacle
Generalization	Students find square number patterns from the given triangle pattern. Then students find the number of small triangles in the 50th term correctly but cannot formulate it in symbols.	Students experience obstacles when formulating regularities from patterns found using symbols.
Manual	Students find multilevel arithmetic patterns and try to solve the problem by adding manually one by one for each term. Even though the pattern found by the student is correct, the final answer given is wrong.	Students experience problems in their calculations, namely errors in calculations or not continuing manual calculations to the term in question.
Formula	Students find multilevel arithmetic patterns but use arithmetic formulas to solve the problem. The mismatch between the formula used and the problem presented makes the student's answer wrong.	Students experience problems in choosing the formula to use, namely using a one-level arithmetic pattern formula for questions that are problems from multilevel arithmetic patterns.

In general, students who worked on task 1 using the generalization process experienced obstacles at the symbolic stage. Students in this category find a pattern of square numbers and then use the regularity in the pattern to find the number of small triangles in the 50th order by multiplying 50 by 50 and getting the correct answer, namely 2,500. However, students do not formulate the regularity in the form of symbols so that students experience obstacles in writing symbols from patterns that have been found. Figure 2 is one answer from a student who experienced this obstacle.



Translation: $1 \times 1 = 1$ triangle, $2 \times 2 = 4$ triangles, $3x$ ×= 9 triangles. Asked how many triangles there are in the 50th order? Answer $50 \times 50 = 2{,}500$. The 50th triangle has 2,500

Figure 2. S1's answer

During the interview, the subject explained as follows. (R represents researcher and S represents student)

R: What do you see in this pattern?

S1: That's 1 then it becomes 1, then 2 becomes 4, if it's 3 it becomes 9. It's like $1 \times 1, 2 \times 2, 3 \times 3$, then the question is the $50th$ term so it is 50×50

Basically, the pattern obtained by subject namely by multiplying n by n itself or in other words it can be written as n^2. However, subjects did not write their expressions in written symbols, but when asked what if they asked the formula for the n term, subjects in this category answered the same thing as subject below.

R: So you have got the pattern, if you ask what the U_n formula is?

S1: What is the U_n formula?

R: You actually already got the pattern, so you can write 50×50, what does the U_n formula mean?

S1: $U_n = n \times n$

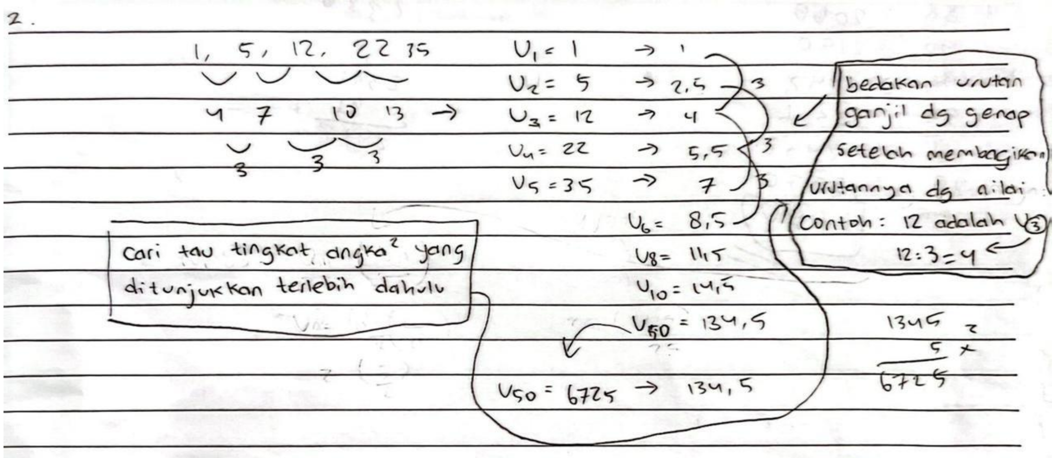
Table 4 presents three categories of students in task 2, along with the strategies they used and the obstacles they encountered while solving the question. It also details the various strategies employed by the students and the challenges they faced in applying these strategies.

Table 4. Analysis of strategies and obstacles learning in task 2

Strategy	Description	Learning Obstacle
Generalization	Students find a pattern in each difference between even and odd terms. Then students try to formulate this regularity. When expressing this pattern, students carry out calculations for which they cannot justify the reasons.	Students experience problems at the expression stage where students carry out calculations in the form of division for each term by trial and error practice based on number patterns. So the form of expression he made was wrong so that the final answer was also wrong.
Manual	Students find multilevel arithmetic patterns and try to solve the problem by adding manually one by one for each term. Even though the pattern found by the student is correct, the final answer given is wrong.	Students experience problems in their calculations, namely errors in calculations or not continuing manual calculations to the term in question.
Formula	Students find multilevel arithmetic patterns but use arithmetic formulas to solve the problem. The mismatch between the formula used and the problem presented makes the student's answer wrong.	Students experience problems in choosing the formula to use, namely using a one-level arithmetic pattern formula for questions that are problems from multilevel arithmetic patterns.

Students who generalize still experience obstacles in the expression stage so they get wrong answers. Apart from that, obstacles were also found in the manual calculations carried out by students. Meanwhile, students who use formulas to solve this problem experience obstacles in determining the right formula according to the context presented.

In general, only one student who worked on task 2 using the generalization process and experienced obstacles at the expression stage. Student who experiences problems in the expression stage but still find the final result even though it is wrong (Figure 3).



Translation: Find out the level of the numbers shown first. Distinguish between odd and even sequences after dividing the sequences by value. Example 12 is U_3 where $12:3 = 4$.

Figure 3. S2's answer

From Figure 3, S2 understands that there is a multilevel pattern where the difference at the second level always increases by 3. However, the student initially only stopped at finding the pattern because she did not remember the formula for the multilevel pattern.

R: So what do you understand from task 2

S2: This is a multilevel pattern, right, and I am confused about what to do because I forgot the formula for this multilevel pattern. So, I tried to find another relationship.

S2 tries to find a relationship for each term where for U_1 the value is 1 then $1:1 = 1$, U_2 the value is 5 then $5:2 = 2.5$, U_3 the value is 12 then $12:3 = 4$, U_4 the value is 22 then $22:4 = 5.5$, and U_5 has a value of 35 so $35:5 = 7$. After that, S2 classifies the terms based on odd and even order terms, where the odd order terms have a difference of 3 from the previous calculation results.

R: Why do you do that division?

S2: I do not know either, just try and see who knows the pattern. Then it turns out that there is a difference of 3 and for each the odd and even order terms. So because what was asked was the 50th term and it was an even number of terms, so I continued to sort them with a difference of 3 until I got the 50th term and that was 134.5. After that 50×134.5 becomes 6,725.

From the results of the interview, it can be seen that S2 has generalized by looking at the relationship between the values he is looking for for even and odd terms. However, the student does not know the exact reason why he is doing the calculations because he is just trying to find the pattern. Therefore, students experience obstacles in carrying out the expression process in task 2, even though students continue their calculations until the final answer, the answer given is wrong.

Table 5 outlines two categories of students in task 3, including the methods they employed and the difficulties they faced in solving the question. It also describes the different strategy used by the students and the challenges they encountered in using this strategy.

Table 5. Analysis of strategies and obstacles learning in task 3

Strategy	Description	Learning Obstacle
Generalization	Students find a pattern in the addition of the first term and the last term. So students can find the sum of the arithmetic series presented. Students succeed through the generalization process of the perception, expression, and manipulation sections.	Students experience obstacles in the symbolic stage. Where students cannot formulate the regularity they find in the form of symbols.
Manual	Students find multilevel arithmetic patterns and try to solve the problem by adding manually one by one for each term. Even though the pattern found by the student is correct, the final answer given is wrong.	Students experience problems in their calculations, namely errors in calculations or not continuing manual calculations to the term in question.

Table 5 shows that there are 2 categories of students in question number 3 along with the strategies and obstacles they experienced in solving the question using the strategy they used. Students who generalize with correct results still experience obstacles at the symbolic stage. Apart from that, obstacles were also found in manual calculations carried out by students.

For solving problem task 3, there was 3 students who solved the problem using a generalization process. Where 2 out of 3 students experienced obstacles at the symbolic stage. Students find a pattern in the sum of the first and last terms of the arithmetic series presented, which indicates that students are going through the perception stage. Students continue the generalization process until the manipulation stage and find the correct answer. However, students experienced obstacles in the symbolic stage similar to those experienced by several students when solving task 1 (Figure 4).

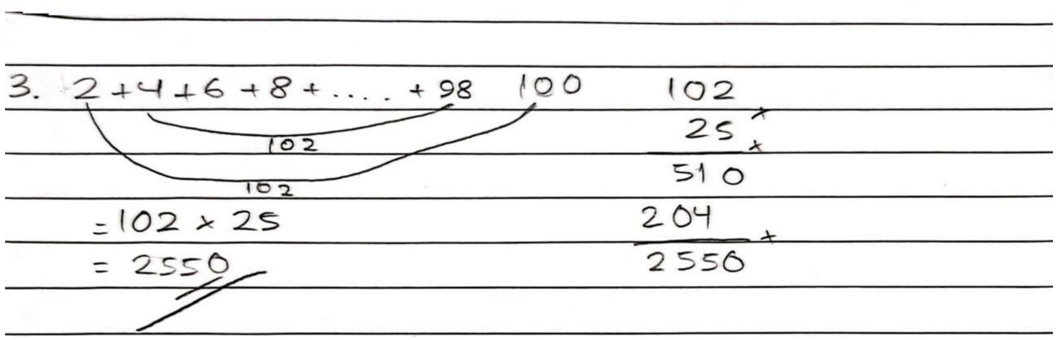


Figure 4. S3's answer

From Figure 4, S3 find a pattern in adding the first term to the last term which was always the same when paired. So S3 continues its calculations from the pattern obtained until it finds the answer, namely 2,550. The answer given by S3 was correct, but S3 experienced obstacles when formulating the pattern he found in the form of symbols. The following are the results of the interview.

R: How you found this pattern?

S3: I actually see this as $2 + 4 + 6 + 8 = 20$, whereas 20 is $10 + 10$ so $2 + 8 = 10$ then $4 + 6 = 10$. So that means 2 to 8 then 4 to 6 there are two pairs of numbers that add up to 10, then I use that pattern, $2 + 100 = 102$, $4 + 98 = 102$. Meanwhile, from 2 to 100 there will be 50 even numbers, so $102 \times 50 = 2{,}550$.

4. Discussion

This study aims to answer the following research questions: (1) What are the specific learning obstacles faced by students in the process of generalization on the topic of number patterns? (2) At which stage of the generalization process do these obstacles primarily occur? The results of the study indicate that students face significant learning obstacles at the expression and symbolic stages of generalization, particularly when trying to symbolically represent the patterns subject have identified.

The research results show, subjects who have gone through the stages of perception, expression, and manipulation correctly, skipped the symbolic stage. The use of symbols or mathematical representations to describe general rules or patterns in number series was not carried out. However, some students choose alternative problem-solving strategies, such as relying on formulas or manual calculations. The strength of this approach lies in its ability to help students reach the correct answer without having to understand the deeper conceptual structure of the pattern. These students demonstrate practical problem-solving skills, often being able to manipulate known formulas and calculations to their advantage. However, skipping the symbolic representation stage can be a disadvantage, as it limits their capacity to generalize in unfamiliar contexts or tackle more complex mathematical problems where symbolic manipulation is essential (Abakah & Brijlall, 2024). Some students may skip the symbolic representation stage, but this does not necessarily hinder their ability to express generalizations (Wilkie, 2024). This is proven by the final result of the subject giving the correct answer even without symbols.

In this case it also shows that students can pass the manipulation stage correctly even though they missed the symbolic stage. The manipulation stage involves the ability to perform mathematical manipulations on symbols or algebraic formulas that describe general rules. The manipulation stage which involves the ability to perform mathematical manipulations on symbols or algebraic formulas is an important stage in the generalization process (Firdaus et al., 2023; Tillema & Gatza, 2017). The weaknesses of students who rely solely on alternative strategies are evident when they face problems that require abstract reasoning and generalization beyond known formulas. Their ability to extend patterns to new contexts is limited, because they lack the symbolic insight to connect mathematical principles abstractly. However, this stage can still be carried out even though students do not go through the symbolic stage, according to Wilkie (2024) every manipulation of an object or process can still be carried out with or without the help of symbols, which include reasoning, naming parts of objects or processes, and pattern reflection.

Subject who has not carried out completely the expression stage then proceed to the manipulation stage but the understanding of what has been obtained at this expression stage is not yet complete. Due to limited understanding, the subject concludes the wrong thing resulting in the final answer being also wrong. This incomplete understanding of the expression stage highlights the significance of learning obstacles in the process of generalization, supporting the need to further analyze how gaps in expression contribute to students' generalization failures. However, the subject's incomplete understanding of the regularity of the problem indicates the need for further development, as stated by Beaton (2014). This is in line with Jackson's (2018) argument that perceptual awareness can lead to skilled behavior, but may not encompass all necessary knowledge. Dicker's (2019) formulation of the problem of perception further underlines the complexity of this process, emphasizing the need for a comprehensive understanding of the underlying regularities.

The findings of this study have important implications for mathematics education, especially in teaching the generalization process on the topic of number patterns. Although the expression and symbolic stages are important parts of mathematical generalization, this study shows that students can still solve the problems presented even when subjects skip this stage. However, this stage should still be mastered by students so that the generalization process can be passed without missing a single stage. As noted in the introduction, the importance of addressing learning obstacles that prevent students from fully engaging with symbolic representation is critical for developing their generalization abilities. Educators must consider the strengths and weaknesses of students who choose alternative strategies such as formula-based or manual calculations. While these approaches provide short-term success, they often fail to provide students with the deeper understanding needed for more abstract problem solving.

Basically, each stage of generalization, namely perception, expression, symbolism, and manipulation, is a series of stages that need to be passed to produce a complete generalization process. The results of this study highlight the need for teaching strategies that provide additional support during the expression and symbolic stages of generalization, helping students develop the skills needed to express general patterns and in symbolic form. These results align with the study's objective to address learning obstacles directly and suggest practical implications for educators in designing didactic interventions to overcome these obstacles.

5. Conclusion

Based on the results and discussion previously described, it can be concluded that there are several learning obstacles in the pattern generalization process. Learning obstacles were found to exist at the expression and symbolic stage which were faced by several subjects differently. Students who experience obstacles at the symbolic stage can generally continue the explanation to the manipulation stage, only skipping the symbolic stage. Meanwhile, students who experienced difficulties at the expression stage also continued to do calculations until they got the final answer, but the answer given was wrong because the expression stage they went through was also wrong. Throughout the assignment, most students did not use the generalization method to explain the problem. Instead, it uses formulas or manual calculations. The obstacle faced by students who use formulas is not understanding the context of the problem so that students can use a different formula from the problem being presented. Meanwhile, the manual method makes students have difficulty calculating. Based on these results, it is recommended that educators implement didactic designs that offer targeted interventions during the expression and symbolic stages, such as using more interactive and symbolic reasoning activities to strengthen students' understanding and generalization skills in number patterns.

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